Using Mathematical Modeling Techniques for Optimized Dairy Herd Management and Decision Making

By

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ABSTRACT

Dairy farm is a complex business enterprise with several uncertain and interacting factors (e.g., biology, environment, market conditions). To become and remain viable in such environment, dairy farm decision makers need to make better-informed decisions. Appreciation of these facts has resulted in extensive on-farm data gathering. However, to obtain useful information for decision-making, data need to be processed. For this purpose, mathematical modeling techniques can be used to develop decision support systems. This thesis applies different mathematical modeling methods- dynamic programming, Markov chain, and Monte Carlo- to evaluate and quantify the economic impact of optimal replacement decisions, reproductive management, and nutritional grouping on dairy herd's profitability. Some of these models were also transformed into decision support systems that can further assist decision-making at the farm level. Dynamic programming optimization and Markov chain simulation were compared to find the optimal replacement decisions in dairy cows. The results showed that although dynamic programming remains the best algorithm for replacement decisions, the simulation method had comparable results. The effect of reproductive management on the herd value was quantified by integrating daily dynamic programming and Markov chain models. The results showed that there is an economic opportunity to differentiate reproductive management strategies according to cows' relative milk productivity. Also, a robust Markov chain was introduced and used for stochastic evaluation of reproductive performance. The study confirmed greater profitability with increased reproductive performance, but a great variation among farms at a given level of reproductive performance was also observed. Finally, a dynamic, finite, stochastic Monte Carlo simulation was developed and used to evaluate the economic impact of nutritional grouping of lactating cows. The results indicated that there was an economic opportunity when grouping homogeneous cows based on both their protein and energy concentration requirements. Regardless of herd size, a maximum relative gain could be achieved by having three nutritional groups beyond the fresh cow group.

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Chapter 1

Introduction

1.1. Background

A Dairy farm is a complex business enterprise (system) with many interacting factors (e.g., genetics, environment, market conditions, and management strategies) that determine the profitability of the business and the amount of the system output. Dairy farmers or their consultants (decision makers) need to make informed and robust decisions continuously (day-to-day) to maintain a sustainable business, given the volatile and uncertain market and environment conditions. Thus, the ability of decision makers to make the right decisions at the right times is an important factor that influences the performance of a dairy herd. Traditionally, decisions are made using intuitive methods, consulting expert knowledge, and using summary statistics from historical records. This approach could lead to static decisions, which could be insensitive to the unstable and frequent changes in market and environment.

Nowadays, dairy farmers have access to large amounts of data that could be used to guide on-farm decisions. However, this historical data could not be used efficiently without further transformations and projections. The raw data need to be processed to obtain useful information and knowledge, which could be used for important on-farms decisions. This step of processing raw data to generate valuable information and knowledge using mathematical models and analysis methodologies is often called "*Business Intelligence.*" Thus the purpose of business intelligence is to provide decision makers with the tools and models required to make effective decisions in a timely manner. Mathematical models are usually abstracted into a computer program that requires data from different sources, and in return generates valuable information in a user-friendly manner to assist decision makers. These interactive computer programs are called decision support systems and should be an integral part of any successful business.

Hence, mathematical models are the cornerstone of the business intelligence and consequently decision support systems. These are indispensable pieces of any well-informed decision making process. Based on the objective and characteristics, mathematical models could be classified into: 1) prescriptive (mathematical programming techniques like linear programming and dynamic programming that find the best allocation of variables that maximize or minimize a predefined objective function), 2) predictive (supervised machine learning algorithms that predict a dependent variable like regression analysis), and 3) descriptive (simulation that help to understand the underlying complex system). These models are usually complex and need to be integrated into decision support systems, which could be successfully used in on-farm decision-making. Thus, decision support systems bridge the gap between mathematical models and dairy farm decision makers to make economically sound decisions.

This thesis aims to evaluates and quantify the effect of some of the most economically influential on-farm decisions (replacement, reproductive management, and nutritional grouping) using mathematical modeling. Consequently, the literature review discusses a subset of mathematical models and their applications in managing replacement, reproductive performance, and nutritional grouping. In this thesis a daily dynamic programming was developed to find the optimal replacement policy and consequently was used to evaluate cow values. Robust Markov chain was introduced to evaluate the effect of uncertain input variables on the output of the model. The last developed model was a terminating, dynamic stochastic Monte Carlo simulation for evaluating the economic impact of nutritional grouping of lactating cows. Moreover, the reproductive management model was transformed into a user-friendly decision support system for on-farm decision-making.

1.2. Thesis Outline

Chapter 2 provides a comprehensive review of the literature on the system analysis, modeling, design, and the process of developing an abstract model of a system. The focus of the chapter is on 3 modeling techniques used in the rest of the thesis. The modeling techniques are Markov chain simulation, dynamic programming optimization, and Monte Carlo simulation. Thus, each section of chapter 2 starts with a detailed general explanation of the model and continues with model's applications in different fields in the dairy industry. The chapter concludes by discussing decision support systems, their importance, and examples in the dairy industry.

Chapter 3 describes a daily dynamic programming model used for finding the value of the cows given optimal replacement decisions. These calculated values were weighted by the herd structure (or proportion of the cows in the state space at steady state) obtained from a daily Markov chain simulation, which produced what we called the "*herd value*". The herd value was further used to evaluate the economic value of different reproductive performance.

Chapter 4 systematically compares the optimal replacement policy obtained from an optimization and a simulation algorithm. Thus, in this chapter, we formally compare the replacement decisions made by a dynamic programming and a Markov chain simulation. The goal was to explore the possibility of using the new straightforward replacement formulation using a Markov chain simulation as opposed to a complex dynamic programming model.

Chapter 5 introduces the robust Markov chain model, which was developed to introduce stochasticity into a standard Markov chain model. The developed model was furthermore used to evaluate the economic and dynamics of different reproductive performances under uncertainty.

Chapter 6 includes a comprehensive explanation of a dynamic stochastic Monte Carlo simulation model that was developed to evaluate the economic value of nutritional grouping. This chapter is divided into two sections: 1) design and validation process of the Monte Carlo simulation model, and 2) Applying the validated model on 5 commercial herds to evaluate the economic benefit of nutritional grouping of lactating cows.

Chapter 7 gives a brief explanation of the developed decision support systems that could assist dairy farmers and their consultants in making better informed decisions.

As seen, this thesis focuses on general mathematical models that could be used in different areas of dairy farm management. The studied and developed models were applied on a few economically important dairy farm decision management strategies. The thesis is not intended to provide a full survey of all the mathematical modeling techniques available for dairy farm decision-making.

Chapter 2

Literature Review

2.1. System Modeling

2.1.1. System Theory and Definition

The application and popularity of system analysis has been on the rise since the published book of Bertalanffy (one of the founders of system theory) entitled “General system theory” in 1968 (Wikipedia, 2014). In summary, general system theory establishes an integrating framework, possibly involving several disciplines, that complex systems could be studied (Dent and Anderson, 1971). Thus, system analysis is a holistic view of a complete system (with their interrelations) with the goal of better understanding.

A definition of a system seems appropriate before delving into its modeling. Different definitions of the term system can be found in the literature. For example, generally system is defined as a group of objects or entities that act and interact with each other towards the accomplishment of some purpose (Banks et al., 2009; Velten, 2009). Therefore, based on the definition, to name a few, there are biological, industrial, and agricultural systems. Agricultural systems can be divided into 4 types of systems: production, enterprise, regional and national, and international and global (Csáki, 1985). Here the focus will be on dairy cattle herds as a specific type of production systems.

2.1.2. System Components

Systems exist in a hierarchical structure, which explains the need of interdisciplinary research. An example of the dairy herd system hierarchy is described in Table 2.1. In a dairy herd system cows are entities that interact with each other in the farm’s physical space (pens) towards a farmer’s goal, which could be overall herd profitability. One level higher the herd system could be a dairy farm with all the cows, machinery, and different crops. Many other subsystems can be at lower levels, which themselves can have more detailed subsystems below them. Higher levels embody the next level of details in the subsystem. Each level of the hierarchy has its own input/output transformation and characteristics and can be used to describe the system. Thus, the point of entry in the hierarchy of any system study depends on the objective of the study and the number of included levels depends on the judgment and requirements of a study (Dent and Blackie, 1979).

Table 2.1. Example of hierarchical structure in a dairy farm system (adapted from Sorensen (1998))

Levels	Systems
N+1	The dairy farm (dairy cows, machinery, crops)
N	**The dairy herd (new born, heifers, lactating cows)**
N-1	The cow
N-2	An organ
N-3	A tissue
…	…

In addition to system hierarchy, systems are also affected by outside factors or “system environment.” Characteristics of the system environment are its unpredictability and variability. System researchers usually establish boundaries around the system environment to facilitate understanding of the system function by restricting the intractable entities and variables; in reality no such boundary exists in any systems (Dent and Blackie, 1979; Banks et al., 2009). Applying this boundary in the studied system is important to modeling the system, since it determines exactly which subsystems must be explicitly represented in the final model structure of the system (Dent and Blackie, 1979).

2.1.3. Why Modeling?

In order to explore the effect of a given change on the system, sometimes it is possible to construct a field trial to investigate the effect of changes on the system’s outcome. However, this is not always possible, especially when setting up the trial is too expensive, time consuming, and highly variable and difficult to control. In these situations, studies of the systems are accomplished through system modeling. In these cases models try to provide an adequate tool to break up the complexity and make the problem at hand more tractable (Velten, 2009). Therefore, researchers usually construct models to understand, analyze, and predict the behavior of complex systems (Gosavi, 2003) through simplification of the system in the built model. Models' usage is not a new idea and is not restricted to the use of computers and generally can be classified into mental, visual, physical, and mathematical models (Ragsdale, 2012). Here the emphasis is on mathematical models, which uses mathematical relationships to describe or represent a system or a complex problem (Ragsdale, 2012). Mathematical models are abstract models and can be represented as equations, functions, and computer programs (Gosavi, 2003).

2.1.4. Modeling Terms

To understand and analyze a system using a model, there is a need to describe different terms used to model a system. The object of interest in the system modeling is called "entity." "Attribute" is the observed property of the entity. "State" of the system is the collection of variables used to describe a system at a given point. An "event" is an occurrence in the system that would change the state of the system. An example of these terms in a dairy herd system follows. A cow could be an entity. Its attributes could be milk yield, body weight, and body condition score. Its state variables could be milk production level (milk production potential), lactation number, and pregnancy status. And its parturition could be considered as an event.

2.1.5. Mathematical Models Classifications

Mathematical models use mathematical relationships and equations to represent a system. Ragsdale (2012) categorized mathematical models based on the degree of knowledge of the functional form of $f(.)$ between the independent variables $X_1, X_2, \ldots, X_n$ and the dependent variable Y and the knowledge about the values of independent variables. Based on this knowledge, the mathematical models can be categorized into prescriptive, predictive and descriptive models. If the $f(.)$ between dependent and independent variables is well-known and the independent variables are under researcher's control the model is called prescriptive. These models include all the mathematical programming techniques such as linear programming, dynamic programming and network models. In the case that the functional form between dependent and independent variables ($f(.)$) are unknown or ill-defined and the independent variables are known and under researchers' control the model is called predictive, such as regression analysis and time series analysis. If the $f(.)$ between independent and dependent variables are known, but the independent variables are unknown or uncertain the model is called descriptive. An example of these descriptive models is simulation. In this thesis, dynamic programming (Chapter 3), Markov chain (Chapter 5), and Monte Carlo simulations (Chapter 6) are studied and applied to dairy herd systems.

2.1.6. Mathematical Programming vs. Simulation

Mathematical programming is a general term used to refer to a broad range of optimization algorithms. Most commonly used technique is linear programming, which is used to find complex planning and investment in different industries and governments (France and Thornley, 1984). A mathematical programming model finds the combination of input variables that yields the optimum (maximum or minimum) output (finding optimum allocations). Thus, the purpose of these models is to answer what to do? questions. Simulation models don't have any optimization algorithm, and the purpose of them is to answer series of what if? questions (Sorensen, 1998).

2.1.7. Simulation

Simulation is a particular type of modeling used to imitate (duplicate) the essence of a system without attaining the actual reality of the system (Wright, 1971; Robinson, 2004). Specifically, simulation is applying modeling techniques to a problem at hand with the objectives of understanding, solving a problem, or answering questions regarding the underlying system (Velten, 2009). Simulation models could be classified by three dichotomies (Law and Kelton, 2007; Banks et al., 2009):

1. Static vs. Dynamic (refers to time dependency of the model)
2. Discrete vs. Continuous (refers to time scale of the system)
3. Deterministic vs. Stochastic (refers to uncertainty of events in the system)

Static models represent the system at a given point in time without considering time changes. On the other hand, dynamic models are the ones that follow the changes in the system through time, thus in dynamic models time is included as a driving variable. This means that in dynamic models, state of the system at t+1 is a function of the state at time t (Sorensen, 1998). Dynamic models, furthermore, could be grouped into terminating or transient simulations (finite horizon) and steady-state simulation (infinite horizon) (Rubinstein and Kroese, 2007; Banks et al., 2009) according to the output analysis. In a terminating simulation a well-specified initial conditions of the system is used as an input and the model runs for some time or a flagging event that stops the simulation. In this type of simulation the modeler wants to explore the evolution of the system over time given the initial condition of the system. On the other hand, in steady-state simulation, the long-run properties of the system are of concern. The model starts with arbitrary state of the system and runs for a long time, that is until the time that the properties of the system is not affected by the initial conditions of the system (Law and Kelton, 2007; Banks et al., 2009).

Discrete simulations are the ones in which the state of the system changes in discrete sets of points in time. Continuous simulations follow the change in the state of the system continuously through time. The choice of discrete or continuous model depends on the objective of the study and the available information about the characteristics of the system (Gosavi, 2003).

Simulation models that produce the same output given a set of inputs are considered deterministic. Thus, input variables in deterministic models are described by their mean values. However, input variables in stochastic models are described by their probability distributions. Therefore, changes in the variance of input variables can change the mean of model's output, whereas the results in deterministic models remain the same (Sorensen, 1998). Stochastic models can, furthermore, be divided into two types: 1) probabilistic models or Markov chain models and 2) Monte Carlo simulation models. In Markov chain models the transition matrix governs the probability distribution of movement from one state to another in the next step. In Monte Carlo simulation models, discrete events are controlled by pseudo-random number generators from appropriate probability distributions related to the events.

As it was discussed, an important property of the models is the hierarchical structure of the underlying systems (Table 2.1). Based on the system's hierarchy, there would be a distinction between empirical and mechanistic models (France and Thornley, 1984; Sorensen, 1998). In empirical models the output of the model relates to the input within the same hierarchy level. On the other hand, a mechanistic model links adjacent levels in a way that parameters estimated at one level can predict the results of a higher level (Sorensen, 1998). Herd level simulation models are typically mechanistic because the herd's production level is indirectly simulated by different milk classes of individual animals (subsystems; (Sorensen, 1998)). These characteristics of simulation models are summarized in Figure 2.1.

Simulation is a two-phase process of building the model and experimentation on it. In this process the real system is replaced by a similar, but abstract, version of the system in a computer to overcome problems related to physical experimentation of the real system (Wright, 1971).

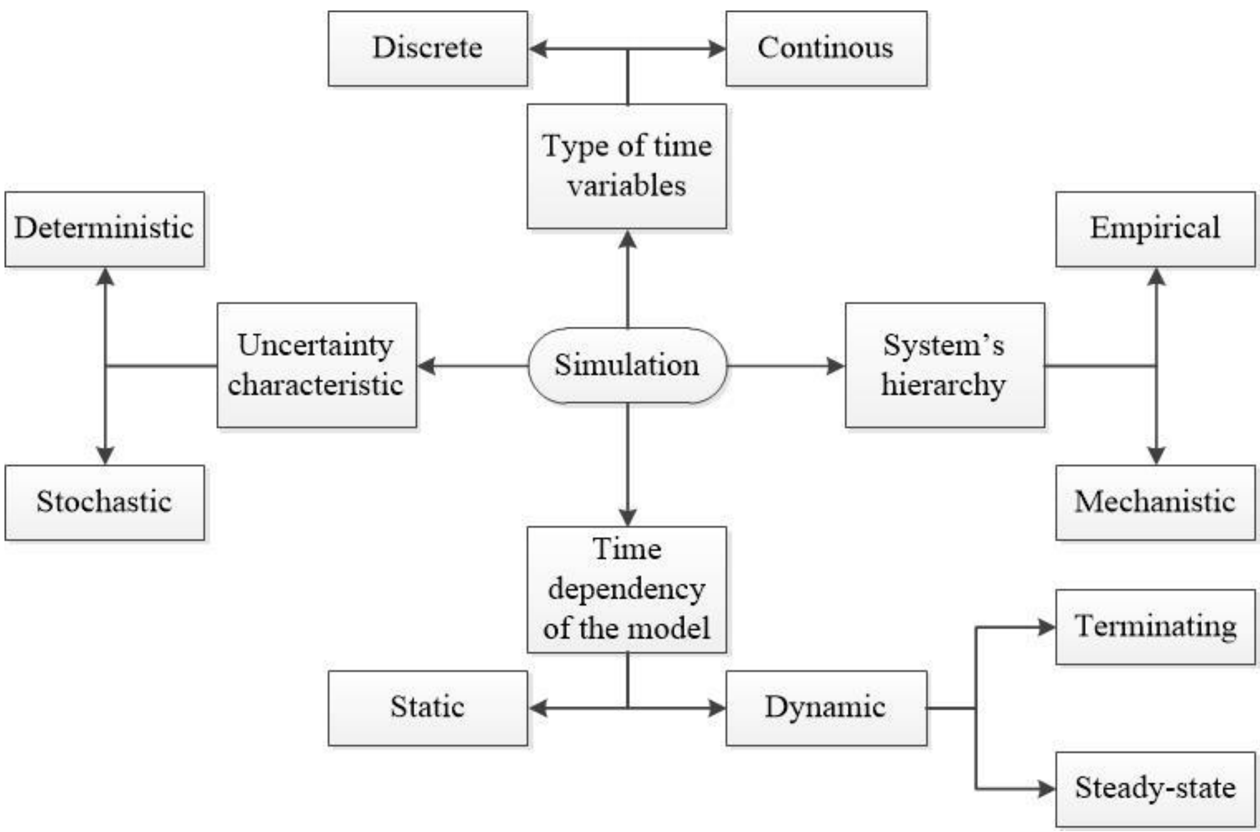


Figure 2.1. Types of simulation (compiled from different sources)

2.1.8. Advantages and Disadvantages of Modeling

There are some considerations that make modeling advantageous over field trials and experimentations on the real systems. The most important factors that make modeling advantageous are related to cost, time, control of the experiments, and ability of comparing different system designs. Experimentation on real system often is expensive and requires a great amount of time to finish. Even in the case that these two factors are not prohibiting the inability to control some aspects of the experiment makes simulation and modeling techniques more appealing in some situations (Robinson, 2004). In addition to a way to estimate the performance of an existing system, modeling can be used to compare alternative proposed system design in a compressed amount of time (Law and Kelton, 2007).

Disadvantages of modeling are the time required to develop the model, data requirements, expertise needed in developing a model, and possibility of overconfidence in the results (Robinson, 2004). The time required to build a full representative model of a system could be dramatic. To develop a representative model of a system, a great deal of "accurate" data from the system is required. Regardless of the quality of the developed model, the input data governs the accuracy and relevancy of the results of the simulation model (this usually referred to as "garbage in, garbage out" phenomena (Chung, 2004)). Overconfidence in the results of a modeling is one of those subtle disadvantageous of simulation. The results from a modeling are as much valid as the method and data used as an input of the model. It is easy to forget about this fact and over-interpret the results of the simulation with a high confidence.

2.1.9. Simulation Study Steps

To construct a successful simulation model from scratch, researchers have suggested different steps (Csáki, 1985; Dijkhuizen and Morris, 1997; Law and Kelton, 2007; Banks et al., 2009). The specific number of steps and the feedback process of these steps could vary among textbooks; however, the main structure of the steps and its iterative nature is the same (Figure 2.2). The first step in this process is the problem formulation and setting up objectives for the simulation model. The simulation process starts up with a problem at hand and sets of objectives that need to be achieved by the simulation model. The designed model greatly depends on the data available on the system, and therefore the second step of simulation study is gathering data and creating useful information that could be used in the model. This step is also concerned with creating a conceptual model (model conceptualization) of the system and all the major points that need to go into the model in an abstract way. Constructing the conceptual model is as much art as science, especially with respect to the ability of the modeler to abstract the essential and important features of the system related to the problem at hand, and making essential assumptions in regards to the problem declared in the objective of the model (Banks et al., 2009). This is done to have a good enough model to answer the questions without dealing with unnecessary details that might clutter the results of the study.

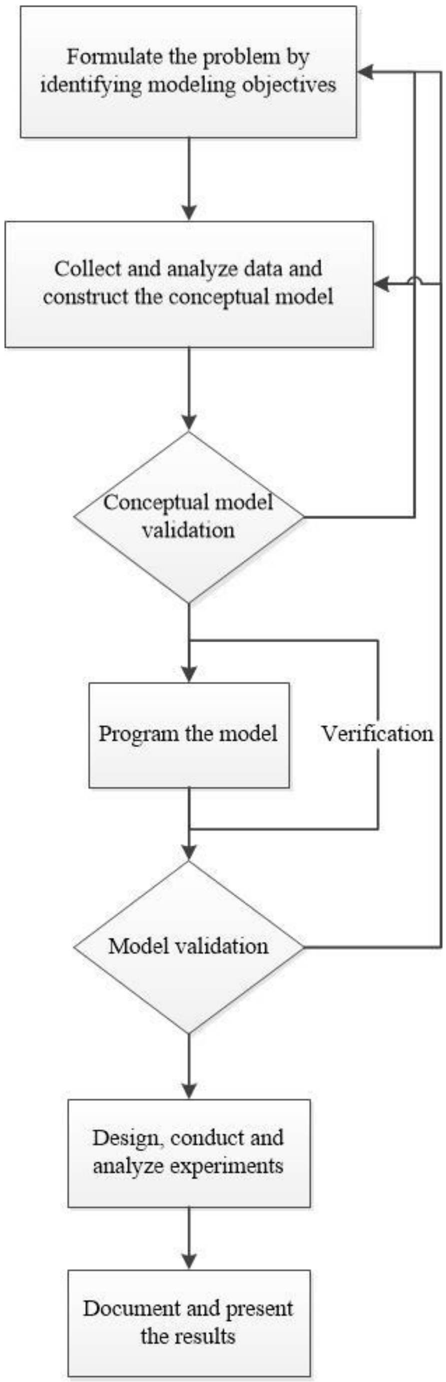


Figure 2.2. Steps for conducting a successful simulation study (adapted from Law (2003))

The best method in creating the conceptual model is with starting up with a simple model and adding in complexities in a stepwise manner (Banks et al., 2009). This would help the process of developing the simulation computer program in the next step. However, before building the computer model a validation is appropriate. This would be a simple sanity check of the conceptual model and its components before going into the computer programming and development phase of the model. After validating the conceptual model of the system, the programming phase of the model starts. Here, the modeler needs to decide what software should be used in building the simulation model. The choice is either using a general programming language (e.g., C++, Java, and Visual Basic. NET), commercial simulation software (e.g., Arena®, SIMUL8®), or spreadsheet modeling using @Risk. Commercial simulation software has the advantage of reducing the required time of programming. On the other hand, general programming languages gives the modeler flexibility in modeling with the expense of time and programming expertise (Law, 2003). Given the choice of a general programming language for the development of the model, choosing an appropriate style of programming is important. Different styles of programming include structural (procedural), object-oriented, and functional programming. The choice of style of programming depends on the simulation method and the time available for the model development. Object-oriented programming approach, due to inherent link to real life objects with the objects in the computer program, has had large attention in the livestock literature (Jørgensen and Kristensen, 1995; Sequeira et al., 1997; Shaffer et al., 2000). In this thesis, a separate chapter has been devoted to the modeling of a stochastic Monte Carlo simulation using object-oriented programming approach to evaluate the economic value of adapting nutritional groupings in lactating dairy cows (Chapter 6).

The programming phase is an iterative process of adding new modules and code snippets, and testing constantly the new units and the overall performance of the modules together. In simulation studies this testing and debugging phase of the computer model is called verification of the model, which checks for correctness of the computer program for performing the simulation and correctly translating the conceptual model into a program (debugging the program) (Law and Kelton, 2007; Banks et al., 2009). After this the developed computer program still needs to go under a validation check. Validation, checks the degree of agreement between the model and the target system (Sorensen, 1990), in other words, it determines if the model is an accurate representation of the system with respect to the study objectives (Law and Kelton, 2007). Usually, validation is considered to be the most difficult and therefore, both objective and subjective methods, are used to test the model (Sorensen, 1990). Objective validation of the model would include statistical tests to find the degree of agreement between the model outputs and the system performance (e.g., goodness of fit tests). However, in practice, it might be unfeasible to perform a field trial in parallel to the model. Thus, as it happens in most livestock models in the literature (Sorensen, 1990), subjective validation techniques are usually used. For the purpose of subjective validation the results from model could be compared with the original system's data, industry averages, expertise opinions, and illustrative graphs from key variables of the system.

After the model is thoroughly validated the researchers need to design, conduct and analyze experiments to answer the questions that have been defined at the first step of the process. The most common approach for running different simulation experiments is to use sensitivity analysis. That is a systematic change of input parameters over a range to explore the effect of a given change on the outcome of the model.

The final step is to document the model, including the conceptual model and the assumptions made throughout the building process, and publish the results obtained from the model in scientific magazines.

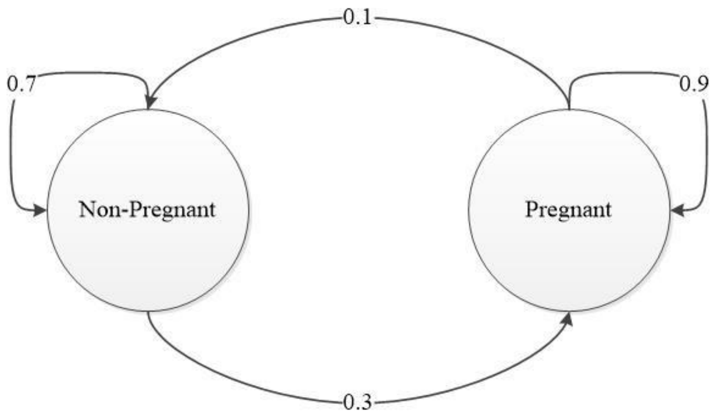
2.2. Markov Chain Simulation Model

Markov chain is a special type of a stochastic process with a key property. Stochastic process's properties change randomly with time. The changing property is referred to as the state of the stochastic process (Gosavi, 2003). For example, a dairy herd system can be considered as a stochastic process, in which states (e.g., milk production and diseases) of its entities (dairy cows) are changing randomly through time (days in milk). The key property of Markov chain can be stated as (Hillier and Lieberman, 1986; Gosavi, 2003; Hardaker et al., 2004):

$$P\{X_{(t+1)} = j \mid X_{(t)} = i\} = f(i,j) \qquad [1]$$

Where $X_{(t)}$ is the system state space at stage (time) t, i is the current state of the system, and j is the next state of the system. This equation states that the probability of being in state j at next stage given the current state of i is constant and equals to the probability of moving between two states. In other words, the conditional probability of any future event, given any past event and the present state $X_{(t)}=i$, is independent of the past event, and only depends on the present state of the process (Hillier and Lieberman, 1986). This is a must condition for the Markov chain models and is called "*Markovian property*" or "*memoryless property*" of Markov chain models.

Beside states variables, Markov chain simulation model is defined by its stage length and the transition probability matrix (**P**). Stage is the time unit between subsequent events in the Markov chain. Elements in the matrix **P** are the probability of moving from each state in the state space to a different or the same state in the next stage (t+1) of the process. Generally, Markov chain represents a system or process that moves between a number of states and the probability of being in different states in this process is governed by transition probabilities. The frequency of changes among the states is dictated by the stage length of the model. A transition diagram of a two states Markov chain process is illustrated in Figure 2.3. In this example, two states are for pregnant and non-pregnant cows in a dairy herd at monthly time intervals. The diagram shows the possible transitions between states and the corresponding probability attached to it with a stage length of a month. Same transitions can be viewed in the form of the transition probability matrix (**P**). An important characteristic of the transition probability matrix is that the summation of the rows across different states should add to one. After all, the rows are probabilities of moving from one state to another in the next stage and they must add up to one.



$$\boldsymbol{P} = \begin{bmatrix} \mathbf{0.7} & \mathbf{0.3} \\ \mathbf{0.1} & \mathbf{0.9} \end{bmatrix}$$

Figure 2.3. Transition diagram of 2 states Markov chain for pregnant and non-pregnant cows with the corresponding transition probability matrix (P)

In this hypothetical and simplified example of a dairy herd system, a non-pregnant cow can either stay in the current state (non-pregnant; 0.7) or become pregnant with the probability of 0.3. The same two way path could be argued for a pregnant cow. A pregnant cow could stay pregnant (0.9) or it could undergo pregnancy loss (0.1). This transition probability matrix is also called one-step transition probability, as it represents the probability of moving from the current state at stage t to another or the same state at stage t+1 (Hardaker et al., 2004), and because the transition probabilities do not change in time, is called stationary transition matrix (Hillier and Lieberman, 1986). The important question in Markov chain models is to determine the state of the system in the future, regardless of the current state of the system. This is usually called the steady-state distribution or equilibrium distribution of a Markov chain and the obtained stabilized probabilities are referred as steady-state or limiting probabilities (π_j) (Agrawal and Heady, 1972; Dijkhuizen and Morris, 1997; Hardaker et al., 2004). The steady-state distribution exists in the case that the Markov chain is irreducible (all states of a Markov chain communicate with each other). The term steady-state probabilities means that the probability of finding the process at state j after a large number of transitions converges to π_j, and is independent of the initial probability distribution defined over the states (Hillier and Lieberman, 1986). This distribution in small problems can be found in two ways. The first way is by multiplying one-step transition matrix (**P**) by itself multiple times until the elements of the result matrix do not change. The second method is to solve a series of linear equations obtained from the transition matrix entries.

The first method to calculate the steady-state distribution is using a one-step transition probability ($\mathbf{P}_1$) for the current stage to find the next stage transition probability ($\mathbf{P}_2$). Therefore, P_2 could be calculated by multiplying P_1 by itself. By multiplying P_2 by itself the transition matrix of the 4th stage ($\mathbf{P}_4$) is obtained. By repeating this process the transition matrix converges to a matrix that does not change from one stage to another, and is called the steady-state distribution ($\mathbf{P}_n$) of the system. Thus, the n-step transition probability matrix can be computed by calculating the nth power of one-step transition matrix (Agrawal and Heady, 1972; Dijkhuizen and Morris, 1997). For example, for a two state process with transition probability as shown in Figure 2.3 the results would look like:

$$\mathbf{P_1} = \begin{bmatrix} 0.7 & 0.3 \\ 0.1 & 0.9 \end{bmatrix} \quad \mathbf{P_2} = \begin{bmatrix} 0.52 & 0.48 \\ 0.16 & 0.84 \end{bmatrix}, \mathbf{P_4} = \begin{bmatrix} 0.3472 & 0.6528 \\ 0.2176 & 0.7824 \end{bmatrix} \dots \boldsymbol{P_n} = \begin{bmatrix} 0.25 & 0.75 \\ 0.25 & 0.75 \end{bmatrix}$$

The obtained P_n is the probability of having non-pregnant (0.25) and pregnant (0.75) cows at a steady-state situation of the system, given the one-step transition probability. In this simple example, the steady-state distribution was obtained after about 25 sequential multiplications of the transition matrices. However, it is obvious that with more states and therefore bigger transition matrices this can be computationally expensive or even impossible.

The second method finds the steady-state distribution by solving sets of equations obtained from the transition probability matrix and the fact that the probabilities across different states must add up to 1. In Figure 2.3 example, the steady-state probability of being at a non-pregnant (π_1) and pregnant state (π_2) will be found as follows:

$$\pi_1 = 0.7\pi_1 + 0.1\pi_2$$

$$\pi_2 = 0.3\pi_1 + 0.9\pi_2$$

$$\pi_1 + \pi_2 = 1 \quad \text{(Normalization constraint)}$$

Here, we have 3 equations and two unknowns and the steady-state distribution could be found using basic algebra. Solving these would give the same results as above, but in a faster and more compact way. In problems with thousands of these equations the equilibrium could be solved in a direct way using Gaussian elimination methods such as Gauss-Jordan method (Tijms, 2003) using available libraries and software packages. However, when the state space becomes very large these direct solution could suffer from computer memory problems (Tijms, 2003).

In real-life dairy Markov chain models the transition matrix is very large (order of hundreds of thousands or millions) and sparse (a matrix with many 0 entries and few non-zero value). Sparsity (also called density is the fraction of zeros in a sparse matrix) is the characteristics of dairy cow systems, in which a few transitions are possible according to age of the cows (Jalvingh et al., 1992). In these models using the matrix notation and solving sets of equations are computationally demanding and for big models even impossible, due to memory issues for storing such big matrices. In these kind of models, the equilibrium distribution is equal to distribution of cows over all possible states in terms of relative numbers of cows, which is governed by the involuntary culling and forced replacements that keep the herd size constant (Jalvingh et al., 1992). This method for finding the steady-state distribution was introduced and used in Jalvingh et al. (1992) and DeLorenzo et al. (1992), and the equations of this method were introduced by De Vries (2004) and were detailed in Giordano et al. (2012). This method is called an iterative approach of finding steady-state distribution (DeLorenzo et al., 1992; Tijms, 2003).

2.2.1. Markov Chain Models in the Dairy Herd Industry

The probabilistic nature of the Markov chain model made it suitable for many problems facing the dairy industry. These types of models could usually be used for projection, economic evaluation, finding distribution of the farm size, and population structure in a long-run (Judge and Swanson, 1962). Markov chain application in dairy industry ranges from health and disease controls (Oltenacu and Natzke, 1976; Sorarrain et al., 1980; Collins and Morgan, 1992; Allore and Erb, 1999; Ivanek et al., 2007), and estimating the herd structure at the steady-state which could be used to explore managerial changes in the herd economics, dynamics, and its environmental impacts. Examples of the Markov chain applications in the literature are: environmental impacts of dairy herds (Cabrera et al., 2006, 2008; Bell et al., 2011, 2013), breeding technologies and reproductive performances (Jalvingh et al., 1993a; Yates et al., 1996; Giordano et al., 2012), add-in module to optimization techniques (DeLorenzo et al., 1992; De Vries, 2004; Kalantari et al., 2010; Kalantari and Cabrera, 2012), and making sub-optimal replacement decisions (Cabrera, 2012b).

2.2.1.1. *Health and Diseases Controls*

Since the 1970, Markov chain model has been used to model the mastitis infection process in dairy cows (Oltenacu and Natzke, 1976; Sorarrain et al., 1980; Allore and Erb, 1999). Markov chain is a fit for mastitis infection due to its sequential structure and natural stochasticity (Oltenacu and Natzke, 1976). In Oltenacu and Natzke (1976) study, each quarter of the udder was considered as the unit of the process and 7 possible infection states were explored. These states were as follows: not infected, clinical and subclinical streptococcus infections, clinical and subclinical staphylococcus infections, and clinical and subclinical infections with other organisms. Their model was discrete despite the fact that the infection process is continuous. The model divided a year in lactation to monthly stages and into lactating and dry months. Because the model was small (transition matrix of 7×7) the model was solved by the matrix multiplication method. Sorarrain et al. (1980) extended this Markov chain simulation by developing both continuous and discrete Markov chain models of the mastitis infection. Their model used the same state variables as the described study above. The results from continuous and discrete results were similar at steady-state and both models were in agreement in the output milk production of the models (Sorarrain et al., 1980). Markov chain model has been also used to evaluate the economic impacts of other diseases such as paratuberculosis and microorganism fecal shedding from dairy cows (Collins and Morgan, 1992; Ivanek et al., 2007).

2.2.1.2. *The Effect of Managerial Changes on the Herd Output*

The main use of the Markov chain model is to describe the dairy herd structure at the steady-state. Due to this capability, Markov chain models have been developed to explore the effect of input parameters on the economic or dynamic of a herd in a long-run. For example, many studies have used the Markov chain model to explore the environmental impact of changes on different input parameters (Cabrera et al., 2006, 2008; Bell et al., 2011, 2013). Cabrera et al. (2008) developed a monthly Markov chain model with 9 parities, 20 months in milk, and 9 months in pregnancy. The model was run for 156 months (till the steady-state) and the herd structure at the steady-state was further used to observe the seasonal manure excretion by dairy cattle (Cabrera et al., 2008). Bell et al. (2011) explored the effect of improving productivity, fertility, and longevity of cows on global warming. For this purpose, using Markov chain model the herd structure was modeled over time. The stage length of 60 days was used and cows were described by using 4 parities status (1 to 3 and 3+) and 10 within-calving interval periods of 60 days (40 states). The authors used the CO_2 equivalent emissions to explore the effect of changes in productivity, fertility, and longevity on the potential impact of dairy systems on global warming. Bell et al. (2013) used the same model to investigate changes in cow production and fitness traits on net income and greenhouse gas emissions in dairy farms. These are samples of Markov chain used in estimating the herd structure in a long run to be used in evaluating the impact of a given change on the variable of interest. More examples of these models in a few publications with a high impact on future researches in dairy industry follow.

One of the earliest monthly Markov chain models, used to simulate dairy herd dynamics, was developed by Jalvingh et al. (1993a). The model was used to study the effect of different calving patterns with the goal of optimization using linear programming in a subsequent study (Jalvingh et al., 1993b). Included state variables in the model were: 15 milk production classes (70 to 130% of average milk production), 10 lactations, 17 months in lactation, time of conception, and month of calving to include the seasonality. Transition probabilities included milk production transition, involuntary and voluntary culling, and probability of pregnancy. This study greatly influenced following studies by describing the model and presenting its power in simulating herd dynamics to explore the effect of management and economic changes on the long-run behavior of a dairy herd. The authors, furthermore, suggested that the model should be integrated with the management information system of the farm in order to gain extra insights before committing to management changes.

St-Pierre and Jones (2001) developed a discrete, dynamic forecasting model using Markov chain model to manage milk production risks corresponding to the unit price and quantity produced (i.e., price and product risk management). The state variables for describing milking cows in the model were 3 parities, 104 weeks in milk, pregnancy status (0,1), and 40 weeks of pregnancy. The state variables for aging heifers were 156 weeks of growth, pregnancy status (0, 1), and 40 weeks in pregnancy. From the state variables it is obvious that the defined stage length in the model was a week. Weekly aging process was followed using transition probabilities in pregnancy, involuntary and voluntary culling, abortion, dry-off, and freshening. The model, furthermore, used Bayes methods to estimate the transition probabilities for the state variables. The authors concluded that Markov chain could be used to successfully represent dynamic of a herd through time, and the fact that forecasted variance, increases monotonically through time in simulation (St-Pierre and Jones, 2001).

The ability of Markov chain to find the steady-state distribution (herd structure) of the cows has also been used in replacement optimization studies (DeLorenzo et al., 1992; De Vries, 2004; Cabrera et al., 2006; Kalantari et al., 2010). In these studies, it is assumed that the optimal culling and insemination policies will affect the freshening patterns and flow of the animals in the herd, which in turns affect the milk production, feed requirements, and replacement needs (DeLorenzo et al., 1992). Thus, to account for these changes in cash flow of a herd, in optimization studies determining the steady-state structure of the herd seems necessary. For example, De Vries (2004) developed a monthly Markov chain simulation module for calculating the herd performance and statistics after finding the optimal replacement policy using dynamic programming. This model used the same state variables as Jalvingh et al. (1993a) except having 12 lactations. The transition probabilities were also the same, but with different values. This model was one of the few models described in detail and was used as a framework by other researchers (Cabrera et al., 2006; Kalantari et al., 2010; Giordano et al., 2012) in dairy management research.

Cabrera et al. (2006) used a Markov chain simulation model to simulate a herd structure to explore the nitrogen leaching of dairy farms under different seasonal conditions. In this model they used a monthly stage Markov chain model with 9 lactations (0=heifers and 1-9 for each lactations), 9 months in pregnancy, and 20 months in milk (and in the case of heifers 32 months of age after birth) as the state variables to describe cows in the model. In this study, a similar approach was used to populate the state space of the cows using probability of pregnancy, and involuntary culling as the transition probabilities and stepping through time to obtain the steady-state herd structure or proportion of the cows at each state.

The stage length of the Markov chain model was reduced to one day in a model developed by Giordano et al. (2012) to study the reproductive and economic impact of different reproductive programs. The model aimed to do a comprehensive comparison among different reproductive programs (100% timed artificial insemination vs. combined heat detection and artificial insemination using Double-Ovsynch protocol), in terms of herd economics and dynamics. Thus, daily stage length was advantageous to create a good representation of dairy herds with respect to reproductive programs. The authors used 3 state variables of 9 lactations, up to 750 days in milk, and 282 days of pregnancy to describe the cows. Involuntary culling, death, reproductive performance, and pregnancy loss were used as the transition probabilities. Using the model authors concluded that as long as the conception rate of a reproductive program is above 30% the combined program would outperform 100% timed artificial insemination program. The two main sources of the generated economic value from reproductive programs were also identified to be income over feed costs and replacement costs.

2.2.2. Summary of Managerial Changes

Following these few examples in dairy industry the common state variables for describing a cow in the models are: a variable representing their age (lactation number or parity), a variable capturing the stage of a lactation (either month, week or days in milk), and a variable that monitors the pregnancy status of a cow (non-pregnant or month, week and days of pregnancy or days open in earlier models). Other variables also have been used to better describe a cow in the Markov chain model. For example, milk production class (DeLorenzo et al., 1992; Jalvingh et al., 1993a; De Vries, 2004; Kalantari et al., 2010), and season of calving (DeLorenzo et al., 1992; Jalvingh et al., 1993a; De Vries, 2004).

Transition probabilities, due to the uncertainty of performance and survival of the cows, can be classified into 4 groups: reproduction, production, disposal (involuntary culling and death), and replacement (Jalvingh et al., 1993a). Based on the used state variables in the specific study one or more of these transition probabilities are used. For example, milk production transition probabilities were used in many studies such as DeLorenzo et al. (1992), De Vries (2004), and Kalantari et al. (2010).

The size of the models discussed above ranges from 3,200 (Cabrera et al., 2006) to more than 600,000 states (Giordano et al., 2012). The overall size of the model directly depends on the number of state variables used to describe a cow in a herd, which indirectly depends on the stage length of the model. The total state space can be found by calculating the Cartesian products of all the state variables dimensions. For example, in De Vries (2004) there would be a total state space of 518,400 (15×12×24×10×12). However, not all of these combinations are possible due to biology, and managerial constraints. An example of a constraint is the relation between pregnancy and month in milk. A cow cannot be 4 months pregnant and 2 months in lactation. After exclusion of those impossible states the total state number of that model by De Vries (2004) was 343,440. The stage length, which indirectly affects the size of the model in these studies ranges from a month (De Vries, 2004) to a day (Giordano et al., 2012).

2.2.3. Sub-Optimal Replacement Decisions

The latest application of the Markov chain simulation has been introduced by Cabrera (2012) in a new formulation of replacement problem and evaluating a cow value in dairy herds. In this monthly model 33 months after calving, 9 months in pregnancy and 10 lactations were used to describe cows in the model. The steady-state distribution of cows (herd structure at steady-state) was obtained by considering the monthly aging of the cows and transition probabilities on involuntary culling, pregnancy, and abortion. Thus, the steady-state herd structure was found like a regular Markov chain simulation by advancing through time and considering the transitions among states (Cabrera, 2012b). However, the idea in this model was to also estimate the net present value of a cow and its replacement for each stage of the model. The net present value of a cow and its replacement were calculated by adding all the economic values at each stage from the start of the simulation until the model reached steady-state. Economic values at each stage were calculated as the sum product of the net revenue of each state and the corresponding herd structure (Cabrera, 2012b). Finally, the cow value was estimated by subtracting the net present value of a replacement heifer from the net present value of a cow and adding the transaction cost (Replacement cost – cow salvage value – calf value) to the result. The calculated cow value further could be used to rank the cows and make culling decisions. Through a systematic comparison the sub-optimal replacement decisions found by this model was compared against optimal replacement decisions found by dynamic programming, which is reported in Chapter 4. The results showed a high correlation between the sub-optimal and optimal replacement decisions (Chapter 4). Thus, this user-friendly model could be used by farmers and consultants to assist them for making on-farm culling decisions.

2.2.4. Introduction to Robust Markov Chain

As described, an application of Markov chain models is to estimate the herd structure, dynamics, and economics after a given change in input parameters (Jalvingh et al., 1993a; St-Pierre and Jones, 2001; De Vries, 2004; Giordano et al., 2012). In addition, the model can also be used to estimate the biological variation among cows in a herd at the steady-state. This variation is due to transition probabilities (probability of pregnancy, culling, death, and abortion), which introduces variation among cows in a herd based on their current state and the chance of moving to another state based on the transition probabilities. However, the Markov chain model does not include uncertainty in the input parameters due to imperfect knowledge. Therefore, the model produces expected value for all the outputs given predefined input parameters (Jalvingh et al., 1992). This refers to the probabilistic nature of the Markov chain model as opposed to the stochastic nature in Monte Carlo simulations, which means that the probabilities used in the transitions are historical expected values, thus ignores the uncertainty around the expected values (Kristensen et al., 2006). To amend this condition in the Markov chains simulation models, a robust Markov chain model could be used. Therefore, a robust Markov chain model was developed and used to assess the economic value of reproductive performance in dairy farms under uncertain conditions (Chapter 5). This method follows the concept of robust optimization used in the operations research literature (Iyengar, 2005). For example, robust dynamic programming considers sets of possible transition probabilities to account for uncertainty in the transition probabilities to capture its effect on the optimal policies being made by the model (Iyengar, 2005). The robust Markov chain model presented in Chapter 5 was envisioned to include such uncertainties and randomness that could be expected within and between targeted dairy farms. In this model, randomness was added to all transition probabilities, milk production levels, and reproductive costs using either of two methods: 1) using a polynomial regression model to build a white noise around the observed historical data for involuntary culling and abortion; and 2) using distributions -such as the normal distribution for milk production levels and triangular distribution for pregnancy rates. Including stochasticity into input parameters (transition probabilities) of a Markov chain model produces uncertainty around the outcomes, both herd economics and dynamics. Presenting the outcome of a Markov chain with uncertainties alongside the farmer's risk preference knowledge could be a useful tool in making better-informed decisions (Olynk and Wolf, 2008). Consequently, this might be helpful to direct their management practices to higher profitability given their current reproductive performance. More specifically, distribution in the outcomes could quantify the probability of reaching a target net return, thus, giving decision-makers a useful cue in directing their management practices to attain higher profitability given their current reproductive performance. These opportunities were not available within a standard Markov chain model.

2.3. Dynamic Programming

In 1957 Richard Bellman published a book entitled "*Dynamic Programming*" which was used for solving sequential decision problems (Kristensen et al., 2006). Sequential decision problems are those that the decisions made in earlier times would affect the future decisions by the decision maker. Dynamic programming (**DP**), also known as Markov decision process (**MDP**) from Howard's 1960 book, is a mathematical technique that utilizes divide and conquer algorithm to divide a multi-stage problem into a series of independently single stage problems to solve it (Puterman, 1994). To define these dynamic decision problems, 5 main elements are needed: actions (or sometimes referred as decisions), stages (or decision epochs), state variables, transition probabilities, and stage returns (also called immediate rewards or costs) (Puterman, 1994; Hardaker et al., 2004). The following notations are based on Puterman (1994) and Hardaker et al. (2004). Except actions and stage returns all other concepts have been described in the Markov chain model section earlier.

2.3.1. Dynamic Programming Terms and Concepts

In the context of DP the total number of time periods that the decision needs to be made is called planning horizon (T) and is divided into decision moments (t) with the predefined and equidistant points in time (stage). The length could be yearly, monthly, weekly, daily, or any other time periods that makes sense for the problem at hand. The planning horizon in DP models could be either finite or infinite, here for simplicity only the finite planning horizon would be covered, but the methods are similar. Typically, at each stage an action ($a_t \in A$; A=predefined sets of finite actions), is chosen by the decision maker. Furthermore, the chosen action at a given stage is affected by the vector of state variables ($S_t \in S$; S=finite state space), used to describe the state of the system. The state changes between two consecutive stages (t-1 and t) are governed by the transformation function (τ_t). Transformation function is a function of the current state, transition probabilities ($P_t(j|s,a)$, and the selected action ($\tau_t=f(S_t,P_t,a_t)$). Transition probability ($P_t(j|s,a)$ denotes the probability that the system is in state j at time t+1, when decision maker chooses action a. In fact, this transformation function is the main difference of the Markov chain model and the DP model. Markov chain's only controlling and governing state transitions is the transition probability matrix. However, DP is basically a Markov chain model with additional control of actions at each stage of the problem, and hence the name Markov decision process (Gosavi, 2003).

In addition, at each stage t there is a stage return or immediate rewards that measures the payoff earned at that stage, and is a function of the current state and the decision made at the stage $r_t=f(S_t,a_t)$. The DP, MDP model, is a particular kind of sequential decision model, in which the available actions, the rewards, and the transition probabilities depend only on the current state and action not on states and actions occupied and chosen in the past (Puterman, 1994). Thus, the qualifier "Markov" is used because in these sequential models the transition probability and stage returns depend on the past only through the current state of the system and the chosen action by the decision maker (Puterman, 1994). This is the "Markovian property" (memoryless property) that has been introduced earlier in the Markov chain models. Thus, these two models have a lot of similarity in the modeling perspective. However, from the solution perspective, due to extra decision step the two models have different solution methods.

2.3.2. Dynamic Decision Model

The sequential decision problem or dynamic decision model is represented symbolically in Figure 2.4. The state of the system at a specified point in time (t-1) is observed by the agent or a decision maker, and based on this an action is chosen. This choice has two consequences: 1) the decision maker receives a stage return (r_t), 2) the system according to the transformation function (τ_t) evolves to a new state. At stage t the similar problem should be tackled by the decision maker, but based on the state and the immediate rewards of that state a different action might be selected (Puterman, 1994). Following this pattern the process evolves through time and the decision maker receives a sequence of rewards based on the selected actions. The action selection at each stage depends on the preselected objective function (performance metric), which could be either maximization or minimization of sequence of rewards or costs. The goal of the process is to find the policy (sets of actions that maps to states; in other words prescription for taking actions at each stage (Tijms, 2003)) that optimize the optimality criterion. Regarding the optimality criterion the decision maker has multiple options. The popular criteria are expected total reward, total expected discounted reward, and expected average reward per unit of output (Puterman, 1994; Kristensen et al., 2006). The choice of the criteria depends on the problem at hand and the planning horizon type (finite or infinite planning horizon). In the remainder of this thesis the total expected discounted reward, which is relevant and most used for livestock problems (Nielsen and Kristensen, 2014), will be used. The total expected reward is a good choice for solving finite planning horizon problems at hand.

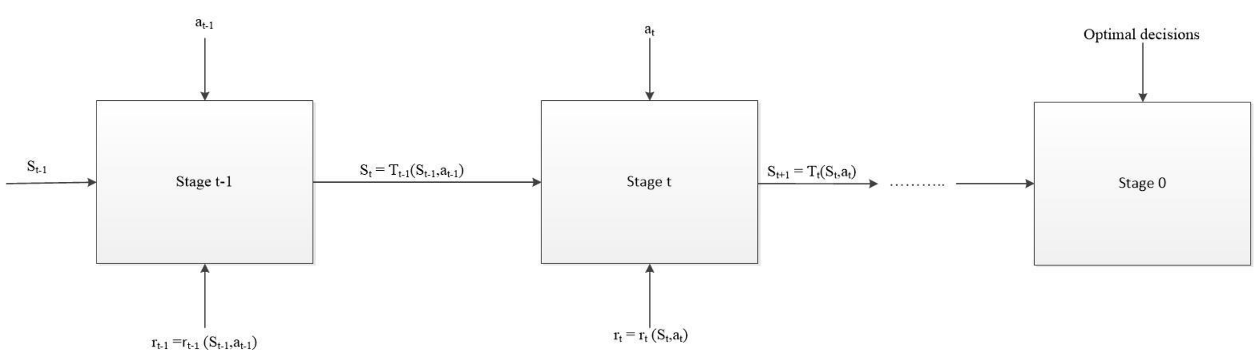


Figure 2.4. Schematic representation of dynamic programming model (based on Hardaker et al., 2004)

2.3.3. Finding Optimal Solutions

Solving a DP problem entails finding the policy (action map from states to actions) vector, and the expected discounted rewards for all the states of the problem. Methods available to find the optimal solutions of a DP problem include: value iteration, policy iteration, modified policy iteration, linear programming, and hierarchical Markov decision process. Only the value iteration algorithm would be described in this thesis, due to its popularity in the literature (both in livestock systems (Nielsen and Kristensen, 2014) and engineering (Alagoz et al., 2015)). This algorithm is straightforward to code in a computer program and it is also the best approach to solve large-scale MDP problems. Both of these factors contribute to the popularity of this method in the literature (Tijms, 2003).

The bellman optimality equation, also known as functional or recursive equations, is as follow:

$$V(s) = \max_{a \in A}\left\{r(s,a) + \lambda \sum_{j \in S} p(j|s,a)V(j)\right\} \qquad [2]$$

Where V(s) is the total expected discounted reward, a is the action to be taken from set of A, r(s,a) is the immediate reward of state s when action a is chosen, λ is the discount factor for considering the time value of money and implies that decision maker has a time preference so that an immediate reward today is preferred over an identical reward at a later stage (Haran, 1997), and $p(j|s,a)$ is the one-step transition probability of going from one state to the next under action a. These are recursive equations and the solving process starts at the end of planning horizon (T) by setting the V(s) of all the states equal to their salvage value. After that, the process continues to the present time and each stage uses the total expected discounted rewards calculated from the previous stages to find the next value, thus the recursive nature of these equations.

2.3.4. Value Iteration Method

The value iteration method which is often simply referred as DP, successive iteration or successive approximation could be used to find the optimal value functions from functional equations (Eq. 2). The functional equation reflects the Bellman's "principle of optimality," stating that an optimal policy has the property that whatever the initial state and the initial decision, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first transition (Puterman, 1994).

Value iteration algorithm steps:

1. (Initialization) Initialize $V^T(s)$ to zero or the salvage value and set T to the end of planning horizon
2. (Value iteration step) For each s in the state space compute V^t (S) by

$$V^t(s) = \max_{a \in A}\left\{r(s,a) + \lambda \sum_{j \in S} p(j|s,a)V^{t-1}(j)\right\}$$

3. (Stopping test) If t = 0 go to step 4, otherwise go to step 2.
4. For each s in the state space choose the action that has the maximum (minimum) of the $V^0(s)$.

This algorithm could be applied to infinite planning horizon case by changing the third step (stopping test). In such case the algorithm would run until the convergence check criterion (ϵ) stops the iterations. The decision found by value iteration are usually not exact and dependent on the choice of ϵ , so called ϵ-optimal decisions (Puterman, 1994). The value iteration method has many different versions used to increase the efficiency of the method, all of which is described in Puterman (1994).

2.3.5. Optimizing Techniques Comparisons

The value iteration algorithm described above is neither the most robust nor the fastest method to find the optimal policy of a problem (Tijms, 2003). In fact, in the infinite planning horizon the value iteration method finds only the approximate (ϵ-optimal) value function and the policies related to that. In this regard policy iteration would be considered to be the most robust method to find the exact solution to infinite horizon problems. Here robust means that the algorithm converges very fast, and the number of iterations is practically independent of the state space (number of states in the model) and varies between 3 and 15 (Tijms, 2003). The algorithm needs to solve S (state space) number of linear equations at each iteration (Puterman, 1994; Tijms, 2003). Thus, this algorithm is not practical for large-scale problems due to computer memory issues (Tijms, 2003). The same problems of computation and memory issues exists in the linear programming solution to the MDP problems, because this method also needs solving S linear equations (Tijms, 2003). For these reasons, policy iterations can only be applied to small size problems (Kristensen et al., 2006). Modified policy iteration combines features of value and policy iterations to increase the efficiency of the policy iteration algorithm for large-scale MDPs (Puterman, 1994), but the increase in efficiency highly dependent on the structure of the model (specifically the number of actions in the action sets and state space size).

In general, the large-scale problems arise in the case that in order to describe a system in the MDP many state variables need to be used. This problem exists in every field; however, it is especially present in the animal replacement models with many sets of variables to describe the variability in the traits and the biology of the animals into the model (Haran, 1997; Nielsen and Kristensen, 2014). This problem in the literature is referred to as the “curse of dimensionality”(Kristensen et al., 2006); and refers to the exponential growth of the state space by addition of state variables into the model. This is a major problem with the policy iteration algorithm, and to a lesser extent for value iteration method, that needs to solve a series of linear equations of size |S| by doing matrix inversion. To solve this issue in large animal replacement problems Kristensen (1987, 1988) developed the hierarchical Markov decision processes. This method combines the computational advantage of the value iteration method with the exactness and higher efficiency of the policy iteration method to solve the “curse of dimensionality” problem in animal replacement problems (Kristensen et al., 2006). This method tries to decompose the state space and reduce the number of states in the MDP, therefore creates a more intuitive way of modeling the replacement problem (Nielsen and Kristensen, 2014). A replacement problem modeled with the MDP model is usually very large due to inclusion of age (lactation and days in milk) of animals as state variables in the model. This inclusion makes a very big sparse transition matrix (a matrix with many zero elements), because only transitions from states at age a to states at age a+1 are possible (Kristensen and Jørgensen, 2000). However, hierarchic Markov decision process models, solves the problem in a special way. This approach does not include age as a state variable, and uses the fact that when replacement occurs, the life cycle of the replacement animal is restarted (Kristensen and Jørgensen, 2000). In addition, due to hierarchy structure, this approach could distinguish between permanent traits and traits that for the same animals vary over time. In the hierarchical MDP the model is split into one main process (founder process) and a series of sub-processes (also called child processes). Then, the permanent traits that are constant over time for the same animals but vary among animals are defined as state variables of the main process and other variable traits are defined as state variables of sub-processes. In this model the main process has the infinite planning horizon and the sub-processes have the finite planning horizon equal to the maximum life span of an animal. Thus, the sub-processes take care of the age as the stage length without the need of considering age as a state variable. The optimization technique of this approach is a mixture of the policy iteration algorithm at the main process with infinite planning horizon for exact, and efficient results, and it uses computational feasibility of the value iteration algorithm for finite planning horizon of large state space at the sub-processes level (Kristensen and Jørgensen, 2000; Nielsen and Kristensen, 2014). For further discussion and algorithm of hierarchical MDP the reference is made to Kristensen (1987, 1988) and the herd management science book (Kristensen et al., 2006).

In contrast to other mathematical programming techniques, there is no standard mathematical formulation of a DP problem. Rather DP is a general type of approach to solve sequential decision problems and the formulation must be made for each particular situation (Hillier and Lieberman, 1986). Therefore, in this thesis (Chapter 3) only the dairy cow replacement formulations are covered.

2.3.6. Dynamic Programming Model in Optimal Dairy Cow Replacement

2.3.6.1. *Characteristics of Animal Replacement Problems*

Many of operational management decisions on dairy farms are naturally sequential and stochastic. Decisions concerning replacement, insemination, and medical treatment are examples of these kinds of decisions on dairy farms. The main differences between animal production and replacement problems compared to industrial problems was summarized by Ben-Ari et al.

(1983). The animal production is a unique problem because of its uniformity (refers to difficulty in defining and measuring traits, and also the high variability in the traits), availability of replacements (standard replacement is not always available), and having a reproductive cycle (optimal time of replacement is influenced by the reproductive cycle). The DP model's sequential and stochastic nature are well suited as a framework for decision support in this area (Kristensen and Jørgensen, 2000), additionally introducing relevant state variables into DP models would be the solution to the mentioned problems in animal replacements (Kristensen et al., 2006). The focus of this review would be on determining the optimal replacement decisions in dairy cows as assets in a production process that needs to be observed periodically (sequentially) and make a decision to whether the current cow state should be replaced immediately or kept for another stage. This decision depends on the criterion of optimality used, net return from the present cow, and net returns from the replacement cows (heifers) (Jenkins and Halter, 1963).

2.3.6.2. *History of Dynamic Programming in Dairy Cow Replacement*

Few years after introduction of DP models in operations research literature the first introduction of the technique in dairy cow replacements was by Jenkins and Halter (1963). However, White in 1959 was the first to introduce and illustrate the technique to solve on-farm decision problems with an application in optimal replacement of laying hens (Kennedy, 1986). After that, the most influential, but unrecognized, study on the dairy cow replacement problem was the dissertation of Giaever in 1966 (Kristensen et al., 2006). This work with 5 levels of lactation, 3 levels of calving interval, and 7 levels of milk yield was an influential work considering the computational power of the time, and mainly due to important considerations regarding Markov property in replacement problems (Nielsen and Kristensen, 2014). Over the years researchers used the technique to determine optimal replacement policies in dairy cows in different countries and using different stage length and state variables. Generally, the state space of models of these studies increased through time, and the stage length decreased. This could be resulted from the available computational power by advances in computer and its availability. There is about 45 studies with a focus on optimal dairy cow replacement in the literature, which has recently been surveyed in a book chapter by Nielsen and Kristensen (2014). Most of the early published works in the literature are solved by the value iteration algorithm. There were few early works that used multiple methods (value iteration, policy iteration, and linear programming) to illustrate each technique. After the introduction of hierarchical MDP in late 80s by Kristensen (1988) this model usage has increased and total of 11 studies in the literature is hierarchical models (Nielsen and Kristensen, 2014). A reason of this rather slow adaptation of hierarchical MDP, even with its efficiency and high performance characteristics, could be attributed to its difficulty in understanding and modeling. Most of the published hierarchical models in the literature used the computer software developed for hierarchical modeling in replacement problems by Kristensen (2003). From a personal experience even with this available software modeling a hierarchical MDP is a demanding task and has a steep learning curve. Here the focus would be on the recent works that had an influence on the current thesis (Chapter 3).

2.3.6.3. *Review of the Models in the Dairy Cow Replacement*

State variables and stage length are important factors in MDP models. These factors determine the size, complexity, and the level of details of the model to be used in the decision making process. There are some commonalities among all the models (normal MDP not hierarchical MDP) in the literature for describing a cow in the model. These state variables are: age of a cow (usually as a lactation or parity number), stage of lactation (in terms of month, week or days in milk depending on the stage length), pregnancy status (as stage of pregnancy or the length of calving interval), and milk yield level of a cow (van Arendonk, 1985b; van Arendonk and Dijkhuizen, 1985; De Vries, 2004). Furthermore, some researchers started to insert the health status of the cows as a state variable into the model for more detailed models. The main health status, studied by the researchers has been mastitis, because of its great impact on the farm economics and replacement decisions (Houben et al., 1994; Bar et al., 2008a; b; Cha et al., 2011, 2014; Heikkilä et al., 2012). Adding extra states for mastitis (and generally health status) adds to the state space of the model, and hence all the mentioned studies, except (Heikkilä et al., 2012), that included mastitis as a state variable used the hierarchical MDP model. Regarding stage length there is a trend towards smaller time intervals by improving the computational power and modeling techniques. The earliest models stage lengths were mostly 1 year (or lactation) and decreased through time. Monthly stage lengths have been the most popular among the researchers, which is mainly due to small computational time even by using the value iteration algorithm. A daily stage length was used in two studies, one using the hierarchical MDP (Nielsen et al., 2010) and the other using the value iteration algorithm (Kalantari and Cabrera, 2012).

In addition, as it was mentioned modeling replacement problem is dynamic and stochastic in nature and therefore needs transition probabilities for different risks involved in the process to take care of evolution of the process in time. These risks and transitions included in different models are mostly derived from the state variables in the model. Most of the studies have included transitions on pregnancy, involuntary culling, and milk yields (e.g., van Arendonk, 1985; van Arendonk and Dijkhuizen, 1985; De Vries, 2004, 2006). In addition, some studies have also included an abortion risk into their calculations (e.g., De Vries, 2006; Kalantari et al., 2010) and others introduced different transition probabilities among mastitis cases (Cha et al., 2014; Houben et al., 1994; Bar et al., 2008a; b; Cha et al., 2011). One of the most important aspect of the series of studies by van Arendonk (1985) on determining optimal replacement policy in dairy cows, is the 15 classes of milk yield and the transitions among different milk classes and the probability of having a heifer entering to the herd with a given milk class. This method was used later by many other researchers to have milk classes in the model and possible transitions among different classes at every decision point (e.g. DeLorenzo et al., 1992; Houben et al., 1994; Haran, 1997; De Vries, 2004; Kalantari et al., 2010). The 15 milk production classes have also been reduced to 5 classes to make a daily DP model more manageable to compare the effect of different reproductive performances on the herd value (Kalantari and Cabrera, 2012).

As it was mentioned DP model uses many economic and probabilistic parameters to find the optimal policy at the individual cow level. Price related parameters include milk price, calf value, carcass value of the culled cow, replacement heifer cost, veterinary cost, semen cost, feed costs, and the market interest rate. Probabilistic risks include involuntary culling, milk production and transition, pregnancy, and abortion. Not all of these parameters have an equal impact on average herd life (representing the average time that a cow stays in the herd) or the replacement rate (voluntary culling decided by optimal decisions) in DP models. Different studies determined that the most important factor affecting replacement rate was the transaction cost for replacement (difference between carcass value and the price of replacement heifer (van Arendonk, 1985b; van Arendonk and Dijkhuizen, 1985; Cardoso et al., 1999; Kalantari et al., 2010)). Average milk production level also has a considerable impact on the replacement in dairy herds (Cardoso et al., 1999; Kalantari et al., 2010). A 20% increase and decrease in milk price and involuntary culling rate had a smaller effect on the average herd life (Kalantari et al., 2010).

2.3.7. Retention Payoff

In the process of determining the optimal replacement policy for dairy cows, MDP (DP) models also create an evaluation of the current dairy cow in the model, given total expected discounted rewards is used as the optimality criterion. Based on the method of calculations the characteristics of this calculated value for every state cow differs. Future profitability, which is the difference between expected net present value of cash flow at the current stage (the value of optimal decision at the current stage) and expected net present value of cash flow for replacement of the cow (van Arendonk, 1985b). The lower bound of the calculated future profitability is zero, which means that the optimal decision is to replace the cow, and the positive values represent the expected profit by keeping the cow until the replacement is optimal instead of immediate replacement. Another way of calculating the cow values is using retention payoff (RPO) as was introduced in De Vries (2004). This value, which is calculated for every cow state in the model, is based on comparing the expected net present value of cash flow from keeping a cow versus expected net present value of cash flow of immediate replacement. The RPO can take negative and positive values. A negative RPO represents the opportunity costs of keeping the cow in the herd until the next decision point, and positive value is the expected value of the cow in the herd (Kalantari et al., 2010). Thus, the RPO (or future profitability) could be used to rank the cows based on their future expected value and be of an assist in culling decisions on dairy farms.

Moreover, because RPO evaluates the value of a cow compared to its replacement it has many other usages than ranking cows for replacement decisions. These examples include evaluating the economic value of pregnancy and abortion (De Vries, 2006), finding the cost of an extra day open (Groenendaal et al., 2004), finding economically optimal voluntary waiting period

(Inchaisri et al., 2011), to evaluate the economic impact of reproduction performance, such as estrus detection and conception rate (Boichard, 1990; Inchaisri et al., 2011, 2012; Kalantari and Cabrera, 2012), economic value of lactational treatments of subclinical mastitis (Swinkels et al., 2005), and economic cost of generic clinical mastitis (Bar et al., 2008a).

The same factors that affect the replacement rate in the DP model also drive the average RPO determined by the model. The average RPO indicates the average value of all the cows in the herd under study. The same change that has a great impact on the replacement rate and average herd life also affects the RPO greatly (herd average milk production, price of replacement heifer, and carcass price). As expected, changes in parameters that increases the replacement rates results in decreases in average RPO of the cows in the herd (Kalantari et al., 2010). Although, all these factors affect the absolute value of the RPO, the important matter for evaluating RPO is to rank the cows compared to their herd mates (Shahinfar et al., 2014), and also notice the RPO trend throughout the lactation (Groenendaal et al., 2004).

2.3.8. A Simple Dairy Cow Replacement Demonstration

A simple example of a dairy cow replacement problem, adapted and modified (using realistic values) from the "herd management science" book (Kristensen et al., 2006), will be presented and solved using the value iteration algorithm. In this example the stage length is a year (or lactation), and cows are described in the model using 5 levels of relative milk production. The milk production is the milk production of each cow state relative to the herd average (Chapter 3). Transition probabilities of changing a milk class from one lactation to another and the probability of a heifer entering into one of the milk classes are based on the assumption of normal milk curves as described in van Arendonk (1985). The original 15 milk classes were reduced to 5 milk classes in Kalantari and Cabrera (2012), which will be used in this simple example. Transition probabilities under two possible decisions of keep and replace are shown in Table 2.2. Transition probabilities under the keep decision shows how likely the cows are going to change milk classes from one year to another. For example, a cow with an average milk class has a probability of 0.452 to stay in the same class, 0.238 to change just one class, and 0.036 to change 2 milk classes (Table 2.2). Cows that are in higher and lower classes have a tendency to the average milk class. Transition probabilities under the replace decision show the probability of a heifer with a specific milk class entering the herd (Table 2.2). The final component of a replacement problem is the expected net revenue of different milk production class under keep and replacement decisions. The net revenue of different milk production classes under the keep decision is assumed to increase linearly from \$1700/cow per year for 76% class to \$3700/cow per year for 124% class (linear factor of \$500 per each higher class). A constant transaction cost (Replacement price – calve value – carcass value) of \$700 was subtracted from the values above to find out the value of new heifers entering the herd.

Table 2.2. Transition probabilities of moving among different milk classes in current stage (year) to different or same milk class in the following stage

Current relative milk class (%)	Transition to other milk classes at next stage under “Keep”					Probability of a heifer at different milk classes under “Replace”				
	76	88	100	112	124	76	88	100	112	124
76	0.302	0.449	0.219	0.029	0.001	0.07	0.24	0.38	0.24	0.07
88	0.117	0.386	0.383	0.106	0.008	0.07	0.24	0.38	0.24	0.07
100	0.036	0.238	0.452	0.238	0.036	0.07	0.24	0.38	0.24	0.07
112	0.008	0.106	0.383	0.386	0.117	0.07	0.24	0.38	0.24	0.07
124	0.001	0.029	0.219	0.449	0.302	0.07	0.24	0.38	0.24	0.07

The value iteration algorithm, as described above, was implemented in the Python language (Python Software Foundation., 2001) to solve the dairy replacement problem (Appendix 1 Script

1). The model was run for 50 stages with a discount factor of 0.85 as it was described in the value iteration algorithm and Eq. 2. The present value of the 5 relative milk classes over 50 stages are presented in Figure 2.5 panel A. It is obvious that the value stabilized after around 30 stages (30 years) in the future. The difference between these present values shows the relative value of each state compared to the others (Kristensen et al., 2006). For example, having a cow with an average milk production (100%) has $865 economic advantage over 12% below average (88%). Another important result that could be calculated from the results is the difference between the keep and replace values (RPO; Figure 2.5; panel B). This value shows the economic advantage of keeping a cow versus replacing it. Thus, whenever the RPO value drops below zero the best decision would be to replace the cow. In this simple example, given the input parameters used, the lowest milk class cow's RPO (76% or 24% below average) was evaluated to -$48, which is the signal for replacement. The decision for other cows would be to keep them until the next decision stage.

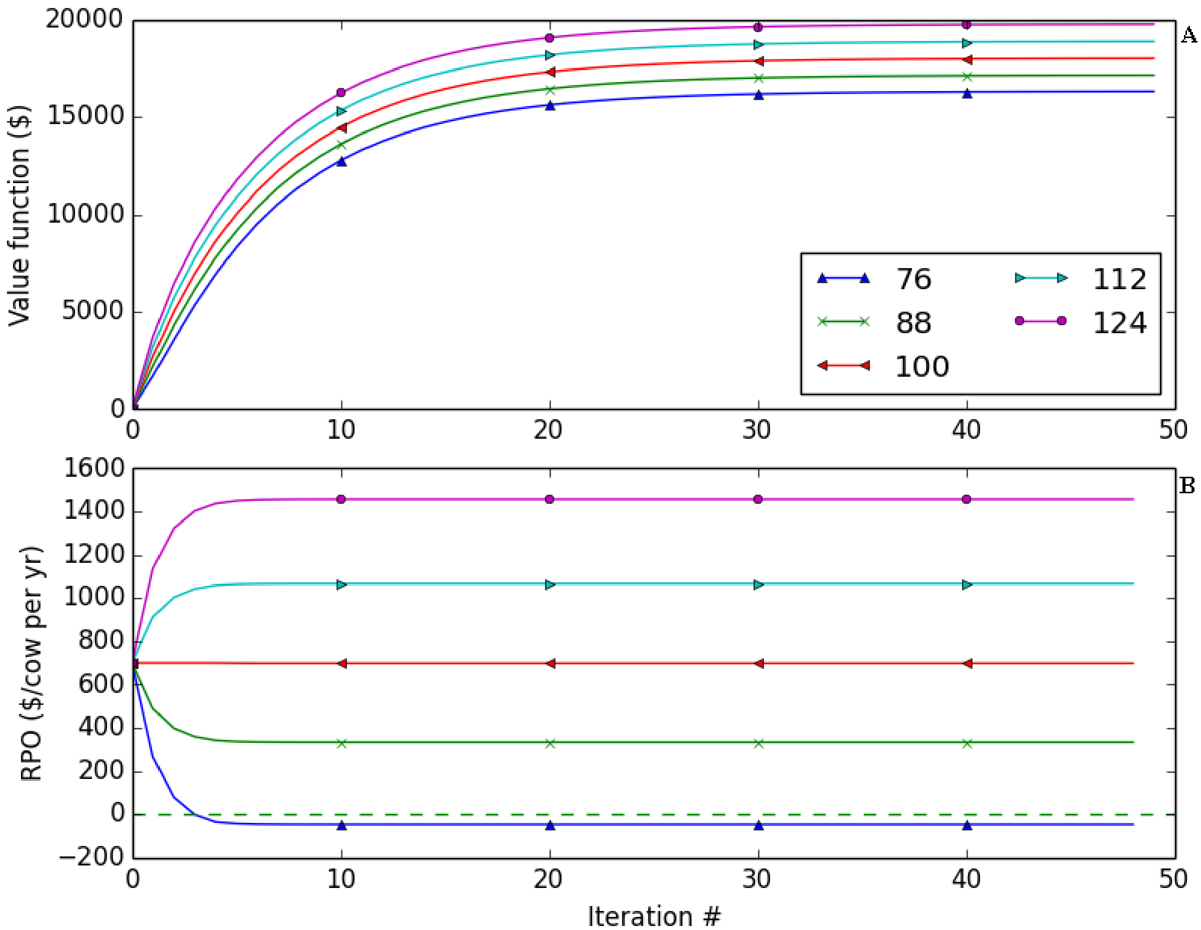


Figure 2.5. Convergence of the present value from value function (panel A), and the calculated retention payoff (RPO = Keep value – Replace value) (panel B) for different relative milk yields.

Should be noticed that this is a simplified example and in real replacement models, besides milk production many other state variables should be considered to describe a more realistic cow in the model.

2.4. Monte Carlo Simulation

Monte Carlo simulation, generally called stochastic simulation or just a simulation, intends to estimate the performance measures of an abstract model built from a given system that is affected by random input variables. This is usually done to obtain a better understanding of the system with respect to decision making under uncertainty of the system. In terms of dependent and independent variables the objective of simulation is to describe the distribution and characteristics of the modeled performance measure (output values; Y), given the distribution and values of the independent variables (input values; X_1, X_2, …, X_k) (Ragsdale, 2012). It models interrelations among the input variables to exploit the uncertainty in input values towards better decision making under risk (Hardaker et al., 2004). Monte Carlo methods were first used to solve physics related problems. In the 1950s, these methods were used in Los Alamos when working on developing the hydrogen bomb and the term Monte Carlo was coined after a well-known gambling house in the area (Brandimarte, 2014). Others (Dijkhuizen and Morris, 1997; Csáki, 1985) have connected the term Monte Carlo with the analogy between games of chance in casinos (e.g., roulette wheel) and the need of random numbers in the simulation.

Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling from specified distributions to compute the distribution of outputs, and have broad areas of applications in different disciplines. Monte Carlo methods could be separated, but not fully, into two techniques. These methods are: Monte Carlo sampling and Monte Carlo simulation. The former term is relevant when the Monte Carlo sampling is used for approximating a numerical integral (often multidimensional and ill-behaved) and some statistical computing (Brandimarte, 2014). A famous example of applying Monte Carlo sampling is to estimate π. In general this method involves random sampling with the goal to estimate a deterministic value (Brandimarte, 2014). Thus, this way of applying Monte Carlo is to solve static and deterministic problems. Monte Carlo simulation, on the other hand, accounts for the dynamics of the systems and entails generating different samples from the model to estimate a probability or an expectation over time (Brandimarte, 2014). These methods are not fully separable because both methods try to estimate a probability or an expectation by generating random numbers, and in principle any dynamic simulation can be considered as the estimation of the integral of a possibly quite complicated function (Kristensen et al., 2006; Brandimarte, 2014). This distinction between Monte Carlo sampling and simulation is not common in the literature. For instance, this has not even been mentioned in some other textbooks (Law and Kelton, 2007; Banks et al., 2009). These textbooks use the Monte Carlo simulation as a general term to refer to both Monte Carlo sampling and simulation. Thus, in this later view Monte Carlo simulation could be either classified as static or dynamic based on the way that the model incorporates the time dimension in the model (Law and Kelton, 2007; Banks et al., 2009). Hereafter, the identifier of static or dynamic is used to define the simulation method.

2.4.1. Monte Carlo Simulation Steps

The building blocks of a typical Monte Carlo simulation following Brandimarte (2014) are represented in Figure 2.6.

1. ***Pseudo-random number generation.*** The cornerstone of any Monte Carlo method is the ability to efficiently generate streams of random numbers from a uniform distribution Ui (0, 1) because the output of system is highly dependent on this first step. Assuming a computer is used to generate random numbers, there are a number of methods that could be used to generate random numbers. The most widely used method in different programming libraries (packages) is the mixed congruential method. The method

generates a sequence of random numbers by calculating the next one from the last one, given an initial random number (called the seed) (Hillier and Lieberman, 1986). Based on the method, the numbers generated should be called pseudo-random numbers (instead of random numbers), due to the predictability and reproducibility of the generated numbers with computers (Hillier and Lieberman, 1986; Law and Kelton, 2007). This fact proves to be useful in developing, debugging, and reducing the variance of the results in the Monte Carlo simulation models (Banks et al., 2009). For brevity, herein, the term random number generation is used.

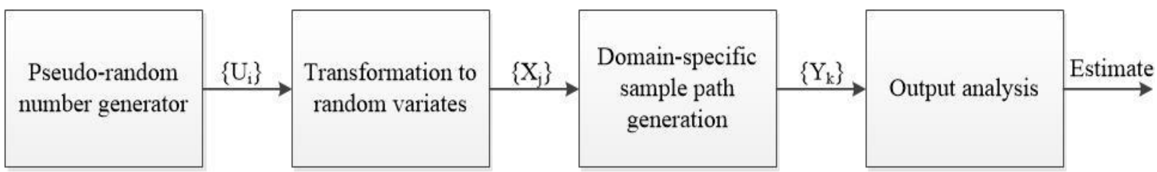


Figure 2.6. Monte Carlo simulation building blocks (adapted from Brandimarte (2014))

2. ***Transformation to random variates.*** Given the sequence of the generated random numbers there is a need to generate random observations from an appropriate distribution. Generating random numbers from well-known distributions (e.g., exponential, uniform, and triangular) or empirical distributions is called random variate generation, and the most basic technique used for this generation is called inverse-transform technique (Banks et al., 2009). Implementing this technique is straightforward, but not always the most efficient way to generate random variates (Banks et al., 2009). Using the inverse-transform technique potentially one could generate random variates from any distribution (Banks et al., 2009; Hardaker et al., 2004). This technique is a method of choice for univariate random variates from continuous distributions when the cumulative distribution function (CDF) is easily invertible (e.g., exponential, uniform,

Weibull) (Devroye, 2006). Even though discrete distributions are not invertible, the same method could be used to generate random variates. The method can be described graphically or by using the equations for specific CDF. A graphical representation of the method in a discrete case for generating random variates from 5 milk classes shown in Table 2.2 is displayed in Figure 2.7.

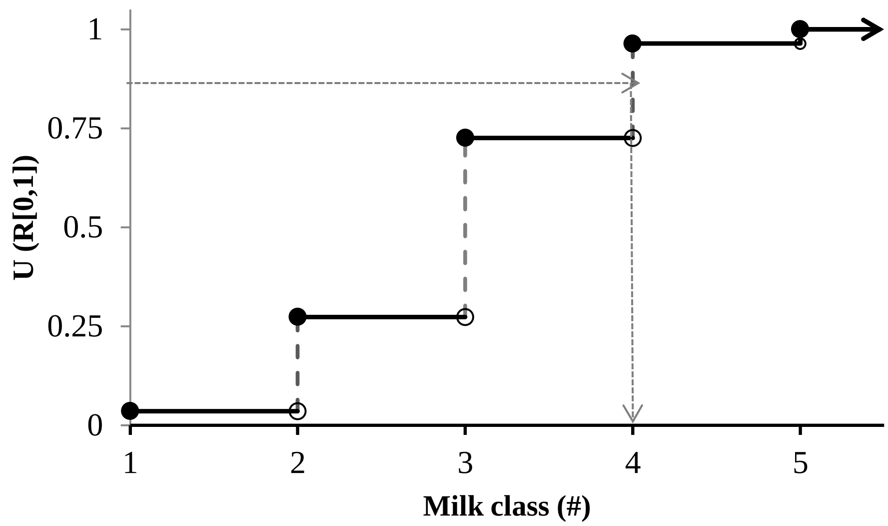


Figure 2.7. Inverse-transform method used to generate random variates from a discrete distribution of milk classes. The cumulative probability of being in 5 milk classes (relative to the average) is determined when a random number is drawn (0.82) its corresponding milk class number is determined from the x axis according to the CDF.

The first step is to plot the CDF of the target function (created from Table 2.2). The second step is to generate a random number U1(0,1). The last step is matching the U1 against the appropriate value on the horizontal axis. In this example by drawing 0.82, the corresponding milk class number is 4. In a continuous case, this translates to setting the inverse of the CDF function equal to U1 and to solve the equation for x. Thus, in both cases generated variates from distributions are in direct proportion to the respective probability of the target distribution function. A similar approach with interpolation can be used for empirical distributions, which is explained in Chapter 6. In this method the shape of the target distribution is not considered for random variate generation, which means that for heavy-tailed distributions this approach uses a large number of samples to generate the target distribution (Hardaker et al., 2004). Techniques used to decrease the number of required sampling in the Monte Carlo simulation are called variance reduction methods (such as Latin hypercube sampling) that will be briefly discussed later.

In addition, in the cases that the CDF is not invertible or computationally expensive and difficult to calculate there are other methods available. For some well-known distributions direct transformation could be used, which does not require sampling. For example, Box-Müller is a method for generating random variates from normal distribution. When the direct methods are computationally expensive or unavailable the method called rejection-sampling could be used. This method uses the probability density function (PDF instead of CDF) of the target distribution. In summary, in this method, an alternative probability distribution (G) is needed, which has an efficient random number generator algorithm and close to the target probability distribution function (F). Then, a random number from G is drawn and is compared to F, which will be accepted only if it falls under the F distribution, otherwise rejected (Rubinstein and Kroese, 2007). The efficiency of this method depends on the closeness of G and F (target PDF). There are other techniques that could be used for generating random variates and the reference is made to the Monte Carlo simulations textbooks (Devroye, 2006; Law and Kelton, 2007; Rubinstein and Kroese, 2007).

There are also some generic methods called Markov chain Monte Carlo (MCMC) for generating samples from any arbitrary distribution and also from multivariate distributions (Kristensen et al., 2006; Rubinstein and Kroese, 2007). The most prominent MCMC algorithms are Metropolis-Hasting and Gibbs samplers (Rubinstein and Kroese, 2007).

3. ***Domain-Specific sample path generation.*** This step is concerned with building a realistic computer representation of the underlying system's characteristics. It is highly related to the conceptual models built from the system, degree of details included, and the objective of the model. Thus, unlike previous steps, this one is specific to the system, the goal of the study, and the problem at hand. It uses sets of input parameters to define and initialize the system and a set of decision rules that govern the behavior of the model with respect to the parameters (Kristensen et al., 2006). For example, to model a dairy cow milk production, body weight, and reproductive status can be used to describe a cow and the cut-off days in milk for breeding can be used as a decision rule. In this step, streams of random variates generated in the previous step ({Xj}) are used to generate different replications of the system. Each replication generated in this step is called the sample path or the realization of the simulated system.
4. ***Output analysis.*** Before using and recommending the results from the simulation verification and validation of the results should be an integral part of every simulation study. Verification checks the conceptual model to be correctly implemented in the model, and consist of debugging and checking the computer implementation. Validation attempts to confirm if the model is an accurate representation of the system (Banks et al., 2009). Compared to verification, validation is more complicated and both objective and subjective approaches are available for it (Sorensen, 1990). Objective validation of the model would include statistical tests to find the degree of agreement between the model

outputs and the real farm performance (e.g., goodness-of-fit tests). However, in practice, it might be unfeasible to perform a field trial in parallel to the model. Thus, as it happens in most livestock models in the literature (Sorensen, 1990), subjective validation is commonly used. Model outputs are contrasted against original data, industry averages, and expert opinions in the field. Visual graphs could also be used to compare the results with expected results.

After checking the model the last step is to summarize the results into estimates, usually in the form of point estimates and confidence intervals. In this step it is important to find out the number of replications needed to gain the degree of precision in the point estimate. Statistics formulas are available to calculate the number of replication needed to obtain a specific precision, which directly relates to the maximum tolerable margin of error in the study (reference on these calculations are made to textbooks (Rubinstein and Kroese, 2007; Banks et al., 2009). Sensitivity analysis could also be used to explore the behavior of the model using different input parameters.

2.4.2. Variance Reduction Methods

Monte Carlo simulation methods are generally flexible and inherently inefficient. As it was discussed in output analysis (and also random variate generation), these methods require a large number of replications to acquire the level of precision needed by the study (Brandimarte, 2014). Generally, the precision of an estimate can be improved by increasing the number of sample size (n; replications). However, this would be an inefficient way to improve the precision of an estimate, because the width of a confidence interval around an estimate decreases with respect to $\sqrt{n}$ (Brandimarte, 2014). This inefficiency in the case of big models makes a difference with respect to the computational time needed to get precise information. Thus, the goal of variance reduction methods is to obtain precise and relevant information at a given number of replications (n) without introducing any bias in the estimator.

These variance reduction strategies could either be tricks to improve the sampling from a distribution or more sophisticated techniques used to synchronize the random number streams of a model for more relevant output analysis (Brandimarte, 2014). The former strategy tries to improve the efficiency of sampling compared to inverse-transform algorithm. Here, just the Latin hypercube sampling among stratified sampling methods will be briefly discussed. In this technique the target CDF is divided into n (sampling size or replications numbers) equi-probable intervals (and hence the name stratified), which will be followed by two step processes of first selecting the interval using a random number and second generating a second random number to determine where within the selected interval it falls (Hardaker et al., 2004). This technique covers two drawbacks of inverse-transform method: 1) the pure randomness of random number generation might not provide a uniform profile from the target CDF, and 2) for getting a precise estimate certain portions of the distribution might carry more weight than other parts (Hillier and Lieberman, 1986).

The second strategy is to use synchronized common random numbers (Rubinstein and Kroese, 2007; Banks et al., 2009). In this method, the same stream of random numbers (common random numbers) is used for the same purposes (synchronized) among different scenarios of the model. This means that same stream of random numbers are used for running different scenarios, which in turn increases the relevancy of output analysis among different scenarios in sensitivity analysis. Thus, it diminishes the need for a large number of replications to reduce the standard deviation of outcomes, which results in more precise estimates. More details on using this technique can be found in Chapter 6.

2.4.3. A Simple Monte Carlo Example in Dairy Cows

A simple static Monte Carlo simulation of dairy cows demonstrates the process. For this purpose the methodological steps as illustrated in Figure 2.2 will be followed. The objective is to find the distribution of annual income over feed costs (IOFC) of the cows in a typical herd with the following assumptions. Annual IOFC is defined as (milk yield × milk price) – (feed cost). To simplify the example it is further assumed that cows are classified into 5 milk production classes (Figure 2.3) regardless of their lactation number. Then, the stochastic inputs are the milk price, milk yield, and the feed costs.

The next step is to analyze relevant data from the past on the parameters in the system to find the input distribution of our uncertain variables. For example, in the case of milk price the historical data from 2013 and 2014 are used to find the distribution of average milk price. Subsequently, different distributions were tried to find the best distribution that can describe the historical distribution of the milk price. Figure 2.8 shows the distribution of historical milk price with the best fitted distribution using Kolmogorov-Smirnov goodness-of-fit test (beta distribution; square error = 0.0223; p-value>0.15; the null hypothesis that milk prices are distributed based on beta distribution cannot be rejected).

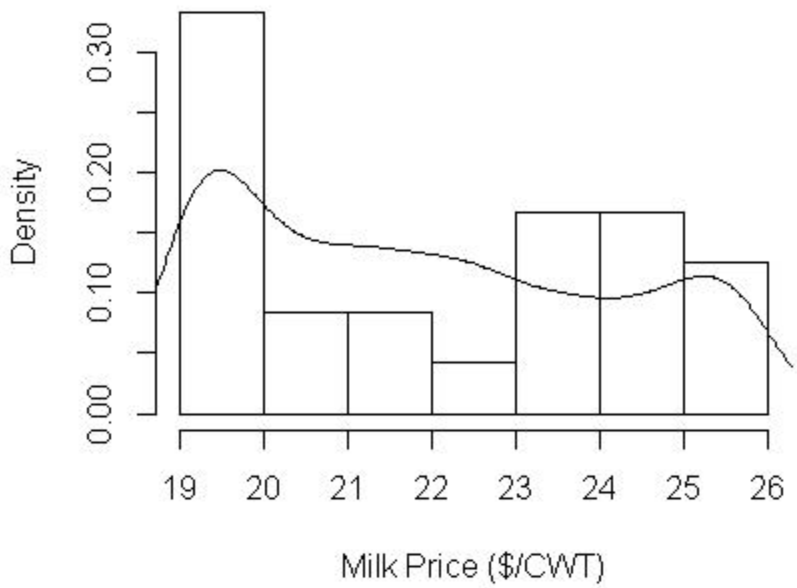


Figure 2.8. Distribution of historical milk price (histogram) and the best fitted distribution on the data (beta distribution expression = 19+7×beta (0.589, 0.825)).

For annual milk yield two scenarios of 5 discrete milk classes as described in Figure 2.7 and continuous milk class from N~(10,000, 1,300) is compared. Finally, the annual feed cost ($/cow per year) is assumed to follow N~(850,30).

The written script in Python (Appendix 1 Script 2) was verified to make sure the program performs as expected. Next, the simulation was run for 1,000 herds of each with 1,000 cows on separate streams of random numbers. This is a hypothetical example and therefore a validation of result is ignored. The cumulative distribution function of IOFC is shown in Figure 2.9. The IOFC of 1,000 simulated cows from one herd under two scenarios of annual milk production (continuous normal vs. 5 discrete milk classes) is displayed in panel A, Figure 2.9. The CDF curve with 5 discrete classes clearly replicates the CDF in Figure 2.7 based on the available discrete probabilities, which demonstrate the Monte Carlo sampling technique (Inverse-transform technique).

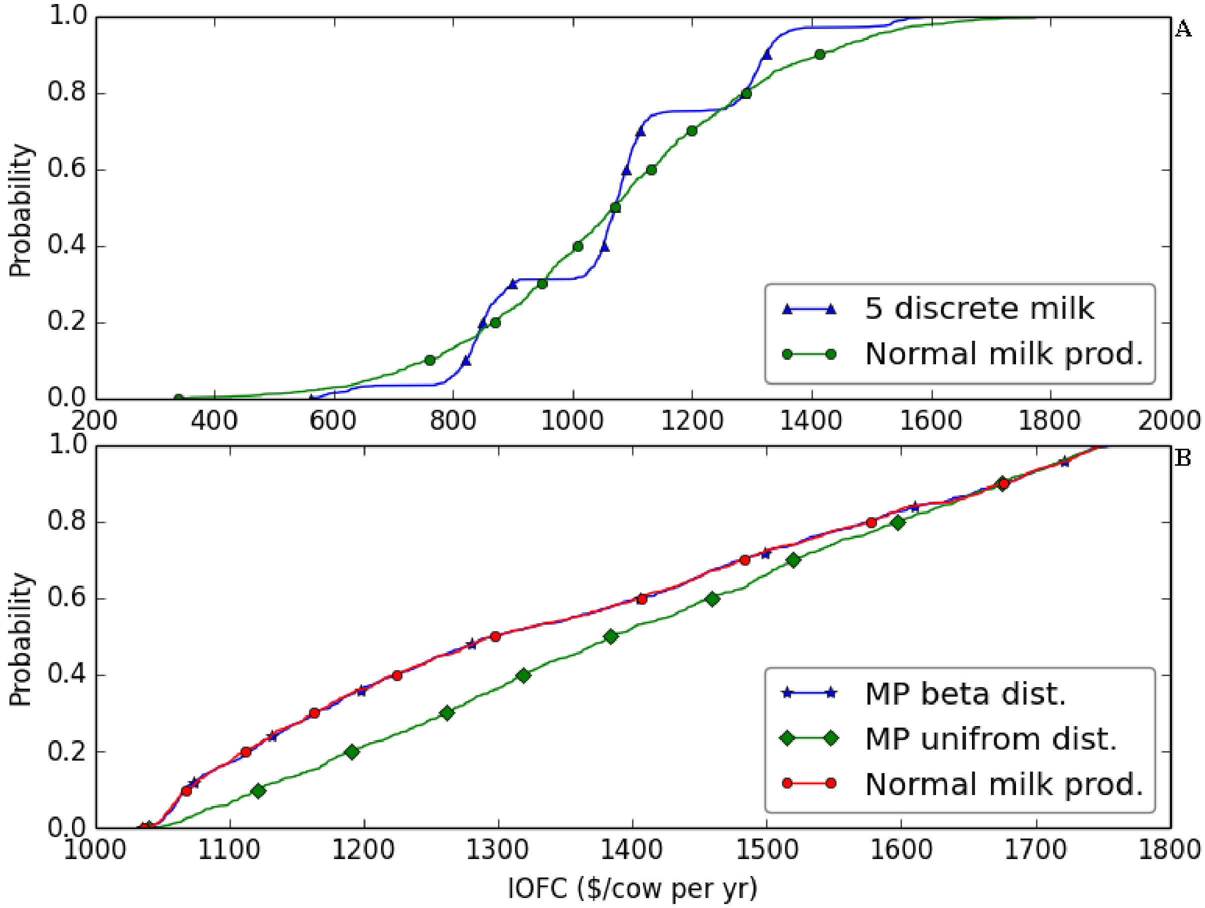


Figure 2.9. Panel A: cumulative distribution function from 1,000 replications of the cows in one herd under two scenarios of annual milk production (5 discrete milk production classes from Table 2.2 vs. continuous milk production from N~(10,000,1,300)). Panel B: cumulative distribution function obtained from averaging 1,000 replications of cows over 1,000 herds under three scenarios (milk price distributed according to beta distribution in Figure 2.8 vs. uniform distribution of milk price U~[19,26] vs. beta distributed milk price when the milk production followed normal distribution described above).

The CDF of the IOFC obtained by averaging over 1,000 cows from 1,000 herds running with three scenarios (comparison between different distribution of milk price (beta vs. uniform) and beta distributed milk price with annual normally distributed milk production) is illustrated in Figure 2.9 panel B. It is clear that when using the uniform distribution for milk price the CDF is linear and it gives the same weight to different milk prices. However, the weights of milk prices are captured when using the beta distribution. The simulation runs under both 5 milk production classes and normal distribution results almost in the same CDF and statistics. The different milk price distribution approaches have an implication in potential decision-making. The average IOFC (±1sd) under beta distribution was 1,339.8 (±220) and under uniform distribution was 1,393 (±198). Using a uniform distribution resulted in an overestimation of the overall IOFC. This is a simple hypothetical example, but it shows the power of the Monte Carlo simulation in sensitivity analysis and in understanding the system under different conditions.

2.4.4. Monte Carlo Simulation in Dairy Herd Industry

Monte Carlo simulation has been used by many researchers (more than 100 studies) in almost every aspect of the dairy herd management to better understand the dairy system, study different management areas of the dairy system (e.g., reproduction, physiology, and genetics), and to explore new emerging technologies and their potential benefits and costs in the dairy industry. Another reason of adapting this technique among dairy researchers from different fields, could be attributed to the accessibility of user-friendly software packages (@Risk in Microsoft Excel or prebuilt models from other researchers DairyORACLE (Marsh, 1986) or SimHerd (Sorensen et al., 1992)) to implement the model in less time and with no or low cost.

The number of studies that have used this modeling technique is staggering and here is classified to better understand their underlying modeling techniques and applications. Models could be dynamic or static. Also, Monte Carlo models in dairy could be classified by the level of simulation in the hierarchy (Table 2.1); cow level simulation vs. herd level simulation. Another classification could be based on the software used for modeling. Pre-built software packages or standalone applications coded by the modeler.

A dynamic, stochastic simulation model developed by Oltenacu et al. (1980) had a focus on reproduction in dairy herds, and was one of the earliest complete herd stochastic models. Later, many dynamic, stochastic simulation models were used to explore the consequences of biological and management changes on the output of dairy systems (Kuipers, 1982; Congleton et al., 1984; Marsh, 1986; Dijkhuizen et al., 1986; Sorensen et al., 1992; Allore et al., 1998b). Most of these early models created a computer model that was subsequently used by the same or other researchers to study different managerial problems on dairy herds. For instance, DairyORACLE (Marsh, 1986), and SimHerd (Sorensen et al., 1992) are among the models used in different studies as the framework.

A brief description of the SimHerd, the most complete herd simulation framework extensively used in the literature, is provided. SimHerd is a dynamic, stochastic Monte Carlo simulation framework with weekly time steps developed to imitate a dairy herd (adults and young stock) to investigate the effect of different management strategies (Sorensen et al., 1992). In this model, 9 state variables were used to describe an individual animal in a herd (age, lactation stage, lactation number, estrus status, pregnancy status, decision for culling, milk production potential, milk production, and live weight). All the discrete events at a cow level were triggered stochastically and governed the herd structure and dynamics. These events included heat detection, pregnancy, fetal death, sex and viabilities of the calf, involuntary culling, and death. Moreover, the herd structure and the production level were controlled by a set of decision variables, which defined a certain production system or a specific management strategy (Sorensen et al., 1992). Since the first version of the SimHerd I (Sorensen et al., 1992) different versions have been released through different research studies. SimHerd II (Ostergaard et al., 2000) added the feeding-health-production complex (potential effect of metabolic and reproductive diseases on the milk production and feed intake) to the SimHerd I framework. SimHerd III (Østergaard et al., 2003) added the risk factors and effects of milk fever to the previous version. The framework was moreover extended by addition of somatic cell count and mastitis to create the SimHerd IV (Østergaard et al., 2005). SimHerd V was developed by separating the genetic and permanent environment of milk yield potential and setting the calf milk yield potential to its parents' average (Ettema et al., 2011).

Monte Carlo simulation has been used to study different aspects of dairy herd systems. Some instances (a subset) of the models in different areas of dairy herd management follows: Different disease controls and its associated decisions and costs (Allore et al., 1998b; Ostergaard et al., 2000; Groenendaal et al., 2002; Kudahl et al., 2007; Steeneveld et al., 2007; Nielsen et al., 2011; Foddai et al., 2014), culling decisions (Marsh et al., 1987; Dijkhuizen and Stelwagen, 1988; Kristensen and Thysen, 1991), evaluating economic traits, choosing selection strategies, and genomic selection (Nielsen et al., 2004; Kulak et al., 2004; de Roos et al., 2011; Ettema et al., 2011; Lillehammer et al., 2011; Axelsson et al., 2013; Hjortø et al., 2015), new dairy technology analysis (Hyde and Engel, 2002; Bewley, 2008), analysis of different parameters related to reproductive performance (Oltenacu et al., 1980; Montaldo, 1996; Plaizier et al., 1997, 1998; Allore and Erb, 2000; Olynk and Wolf, 2008; Inchaisri et al., 2010; Brun-Lafleur et al., 2013; Galvão et al., 2013; Rutten et al., 2014), and nutrition and feeding systems (Pecsok et al., 1992; Williams and Oltenacu, 1992; St-Pierre and Thraen, 1999). The models listed above are all stochastic Monte Carlo models, however; some of them are static (no time inclusion) and others dynamic (time as a variable in the model). A brief list of studies with some of their characteristics is summarized in Table 2.3.

Table 2.3. Brief modeling characteristics and application of subset of stochastic simulation studies in dairy industry (chronologically ordered).

Study	Modeling characteristics[1]	Used software	Application
Oltenacu et al. (1980)	Dynamic, herd, day	GASP[2]	Reproductive process
Kuipers (1982)	Dynamic, herd, month	Coded[?]	Comparing selection and culling decisions
Bailie (1982)	Dynamic, herd, month	Coded[?]	Breeding management efficiency
Congleton et al. (1984)	Dynamic, herd, year	GASP[2]	Determining the profitability of extending cow herd life
Marsh (1986)	Dynamic, herd, year	Coded-C (Developed DairyORACLE)	Evaluation of managerial changes (reproductive, health)
Dijkhuizen et al. (1986)	Dynamic, herd, 20 days	Coded-Fortran 77	Economic evaluation of management decision with respect to production, reproduction, and culling,
Marsh et al. (1987)	Dynamic, herd, year	DairyORACLE[3]	Economic evaluation of 4 culling policies
Dijkhuizen and Stelwagen (1988)	Dynamic, herd, 20 days	Coded-Fortran 77	Economic evaluation of 4 culling policies
Skidmore (1990)	Dynamic, herd, year	Coded-Fortran 4.1	Evaluation of managerial changes (reproductive, production)
Sorensen et al. (1992)	Dynamic, herd, week	Coded-Turbo Pascal (developed SimHerd I)	Evaluation of different feeding regimes, and different culling and reproductive strategies
Schrooten and Van Arendonk (1992)	Static, cow	?	Genetic improvement with Multiple Ovulation and Embryo Transfer (MOET)
Sørensen et al. (1993)	Dynamic, herd, week	SimHerd I	Effect of different dry period lengths
Ostergaard et al. (1996)	Dynamic, herd, week	SimHerd I	Estimation of technical and economic effects of using one vs. multiple TMR feeding groups
Plaizier et al. (1997)	Dynamic, herd, week	SimHerd I	Relationship between measures of reproductive performance and net revenue
Allore et al. (1998b)	Dynamic, herd, variable based on events	Coded-C/C++ (developed SIMMAST)	Effect of mastitis on composition of bulk tank milk

Study	**Modeling characteristics**[1]	**Used software**	**Application**
St-Pierre and Thraen (1999)	Static, cow	?	Estimating optimum allocation of NE_L and CP
Allore and Erb (2000)	Dynamic, herd, day	SIMMAST+SIMHEALTH (developed DairySim)	Evaluating the effect of extending VWP by 100 days on disorders and health issues (e.g., mastitis, ketosis, milk fever, dystocia, retained placenta)
Bargh (2000)	Dynamic, cow, day	ASCL[4]	Predicting the nutritional effects on milk fatty acid profile
Ostergaard et al. (2000)	Dynamic, herd, week	Developed SimHerd II	Evaluation of managerial changes in feeding, health and production
De Vries (2001)	Dynamic, herd, day	Coded-C++ (developed DASIMO)	Studying statistical process control charts in simulated dairy herds
Groenendaal et al. (2002)	Dynamic, herd, 6 months	VBA + Microsoft Excel	Evaluation of economic and epidemiological impacts of different control strategies for Johne's disease
Hyde and Engel (2002)	Static, cow	@Risk add-in Microsoft Excel	Calculating break-even value for robotic milking systems
Østergaard et al. (2003)	Dynamic, herd, week	Developed SimHerd III	Evaluating the long term effect of control strategies against milk fever
Sørensen and Østergaard (2003)	Dynamic, herd, week	Developed SimHerd III	Analyzing the economic consequences of postponed first insemination with different reproductive performance
Nielsen et al. (2004)	Dynamic, herd, week	SimHerd III	Driving economic values for different traits
Østergaard et al. (2005)	Dynamic, herd, week	Developed SimHerd IV	Evaluating different pathogen-specific mastitis control strategies
Ettema and Østergaard (2006)	Dynamic, herd, week	SimHerd III	Evaluating prevention and control strategies of clinical lameness with its economic impacts
Steeneveld et al. (2007)	Static, cow	@Risk add-in Microsoft Excel	Economic effects of treating chronic subclinical mastitis

Study	Modeling characteristics[1]	Used software	Application
Bewley (2008)	Static, cow	@Risk add-in Microsoft Excel	Examining technical and economic feasibility of automated body condition scoring
Olynk and Wolf (2009)	Static, cow	@Risk add-in Microsoft Excel	Economic and risk analysis of artificial insemination with Ovsynch and Cosynch protocols
Bruijnis et al. (2010)	Dynamic, herd, month	@Risk add-in Microsoft Excel	Economic effects of foot disorders in dairy cattle
Inchaisri et al. (2010)	Dynamic, herd, week	@Risk add-in Microsoft Excel	Evaluating economic consequences of reproductive performance scenarios
Inchaisri et al. (2011)	Dynamic, herd, week	@Risk add-in Microsoft Excel	Analysis of economically optimized voluntary waiting period
Sørensen et al. (2011)	Dynamic, Cow, week	ADAM[5]	Effect of using sexed semen on genetic gain in commercial herds
de Roos et al. (2011)	Static, cow	?	Effect of genomic selection on genetic improvement and inbreeding
Ettema et al. (2011)	Dynamic, herd, week	Developed SimHerd V	Identifying the importance of genetic progress in milk yield when evaluating different reproductive strategies
Weigel et al. (2012)	Static, cow	?	Quantifying the genetic gains of using genomic testing on replacement heifers
Galvão et al. (2013)	Dynamic, herd, day	Netlogo[6]	Comparing economic outcome of different reproductive programs
Brun-Lafleur et al. (2013)	Dynamic, herd, day	?	Reproductive process sensitive to milk yield and body condition score
Rutten et al. (2014)	Dynamic, cow, week	@Risk add-in Microsoft Excel	Evaluating the return on investment of activity monitors for better estrous detection
Yin et al. (2014)	Static, cow	QMSim[7]	Comparing genetic gain and inbreeding coefficients of dairy cattle using natural service bulls

Study	Modeling characteristics[1]	Used software	Application
Hjortø et al. (2015)	Dynamic, herd, week	SimHerd IV and ADAM	Evaluating genomic testing's interaction with reproductive surplus

[1] Model characteristics including time dependency (dynamic vs. static), simulation level (herd vs. cow), and time steps in dynamic simulations

[2] General Activity Simulation Program is specialized purpose simulation language

[3] With some modifications from the original DairyORACLE (Marsh, 1986)

[4] Advanced Continuous Simulation Language used for modeling and evaluating performance of time-dependent continuous systems

[5] ADAM is a computer stochastic simulation package written in FORTRAN 95 for modeling selective breeding schemes (http://adam.agrsci.dk/)

[6] A multi-agent programmable modeling environment (https://ccl.northwestern.edu/netlogo/)

[7] QTL and Marker simulator for simulating large scale genotype data (http://www.aps.uoguelph.ca/~msargol/qmsim/)

[?] The programming language is not known

All of these models follow the building blocks of the Monte Carlo simulation as described in Figure 2.6. However, what makes the structure of these models different among studies is the domain-specific knowledge representation, which models the sample paths or realization (step 3 in Figure 2.6). Common state variables to describe a cow in these studies were traits related to production (e.g., milk production potential, %fat, and %protein), reproduction (e.g., gestation length, days dry, and days open), nutrition (e.g. DMI, nutrient requirements), and age related traits (e.g., age, lactation number, DIM). However, some studies had extra state variables to describe exactly the problem to be solved by the model. For example, a model developed by Allore et al. (1998) used extra state variables to describe mastitis cases related to different pathogens and the treatments that the cow received. Furthermore, in dynamic models a cow proceeds through time steps (e.g., monthly, weekly, or daily), which would trigger different events in the model. In turn, these events update cow's characteristics. Events used in the models are also similar and relates to mainly reproductive cycles of a dairy cow. These events include: calving, ovulation, estrus detection, service, conception, and abortion (De Vries, 2001). Other events relate to the involuntary culling, voluntary culling, and mortality, which alongside the reproductive events shape the herd structure. Another factor in these simulations is input prices and costs. These factors vary dramatically among studies, and usually are set to some default values instead of drawing them randomly. Keeping some factors deterministic is a usual act in stochastic models, to focus on important aspects of the system to be studied without unnecessarily cluttering the results.

In the following sections some of the studies with different application in the dairy industry are reviewed. The emphasis is on dynamic stochastic models that were used as a reference in Chapter 6 of this thesis.

2.4.4.1. *Health and Disease Control*

Many studies have used simulations to obtain extra knowledge about specific treatment or control of dairy cow diseases. Mastitis has been the subject of many studies (Allore et al., 1998a; Østergaard et al., 2005; Steeneveld et al., 2011; Hagnestam-Nielsen and Ostergaard, 2009), which are used as an example of disease models. A stochastic computer model named SIMMAST was developed, validated (Allore et al., 1998b), and subsequently used to evaluate different strategies to control mastitis (Allore et al., 1998a). The SIMMAST is a daily step dynamic, stochastic simulation model written in the C language to simulate intramammary infections caused by different strains of pathogens and strategies to lower the somatic cell count of the bulk tank (Allore et al., 1998b; a). The SIMMAST was run for 2 years under 7 possible combinations of mastitis preventive strategies, lactation therapy, and dry cow antibiotic therapy

(Allore et al., 1998a). The results showed that no single factor strategy was dominant and the best results were achieved using a combination of scenarios (Allore et al., 1998a). Few years later in a similar study Østergaard et al. (2005) used the SimHerd IV with weekly steps to run an extensive sensitivity analysis with more pathogens and mastitis types than Allore et al. (1998a) study. Because the model was implemented on top of SimHerd framework, it was able to account for the interaction between different management strategies (e.g., reproduction, culling) and mastitis control strategies at the herd level which was not possible in previous models (Østergaard et al., 2005). This model was further used to examine the economic impact of reduction in the incidence of clinical mastitis in dairy herds (Hagnestam-Nielsen and Ostergaard, 2009). Economic impacts of treatment chronic subclinical mastitis caused by one pathogen were estimated in a static stochastic simulation using Microsoft Excel @Risk add-in software (Steeneveld et al., 2007). Comparing to previous models, this model was much simpler, less detailed, and at the cow level (as opposed to herd level in the previous study).

Other diseases have also been studied in the literature such as Johne’s disease control (Groenendaal et al., 2002), milk fever (Østergaard et al., 2003), bovine viral-diarrhea (Viet et al., 2004), and foot disorders (Bruijnis et al., 2010).

2.4.4.2. *Culling Decisions*

Culling decision has a great impact on farm’s profitability. Dynamic programming (explained in section 2.3) could be used to find the optimal economical replacement policies in herds. However, to be able to compare different practical culling policies in dairy farm stochastic simulations have been used (Marsh et al., 1987; Dijkhuizen and Stelwagen, 1988). Both models used 4 different culling strategies to cover a variety of insemination and culling policies that were practical. Two policies were based on a single predefined cut-off point of service (165 and 250 days after calving), and the other two more restrictive options varied according to the milk production level and stage of lactation (different combination of milk production level and days after calving (Marsh et al., 1987)). Similar policies with different cut-off points and milk production level and stage of lactation, including some optimal policies found in a DP model were used in another study by Dijkhuizen and Stelwagen (1988). For the comparisons Marsh et al. (1987) used a previously developed dynamic, stochastic simulation model, DairyORACLE (Marsh, 1986). The results showed that for maximum profit the culling policy should not be too restrictive. Although, these strategies decreased the calving interval, they increased the replacement rate and therefore had a negative impact on the profitability. The study suggested that, in the US, cows should be bred until 250 days after calving as long as the low producing cows were culled for low production irrespective of reproductive status (Marsh et al., 1987).

Similar dynamic, stochastic simulation study was conducted in the Netherland with similar 4 culling policies as above (Dijkhuizen and Stelwagen, 1988). They concluded that at <50% heat detection and < 40% conception rate, there is no value of culling the cows based on the combined policy based on poor productive and reproductive performance. However, they found that in better performing herds making culling policies based on a more restricted measure of production and reproduction adds to the profitability of a dairy herd (Dijkhuizen and Stelwagen, 1988).

The Monte Carlo stochastic simulation approach has been used to calculate the total expected net returns during the next year and that value was used for ranking animals. Kristensen and Thysen (1991) compared the decisions being made by DP and stochastic simulation and reported insignificant difference between the two models.

2.4.4.3. *Animal Breeding and Genetics*

Studies in animal breeding and genetics, using stochastic simulation, can be divided into 2 types: 1) evaluating the economic value of important traits 2) evaluating the value of different selection strategies.

Nielsen et al. (2004) and Kulak et al. (2004) used the SimHerd dynamic simulation model to evaluate the economic values of different production and non-production traits under different farm scenarios. The scenarios were different based on the herd size, level of the trait, prices, and presence of milk quota. Energy corrected milk, conception rate, mastitis, body weight, and involuntary culling are examples of the traits included in both studies. The evaluated economic value for all the traits related to diseases was negative, and the absolute value depended on the severity of the cases.

Since the availability of the genomics selection many different studies have explored the ways that this technology could potentially improve genetic gain (due to selecting accurately young cows that lead to shorter generation interval) and its effect on inbreeding compared to progeny testing in dairy cows (de Roos et al., 2011). For this purpose a closed nucleus herd with an annual birth number of 1,000 males and females were simulated under two selection criteria of progeny test and genomic selection (de Roos et al., 2011). Running static, stochastic simulation the study found that genomic selection would increase the rate of genetic gain (+30%/ yr compared to progeny test) and decrease the rate of inbreeding per generation (de Roos et al., 2011). Similar results regarding the better genetic gain in genomic selection at lower rate of inbreeding was found in another study using static, stochastic Monte Carlo simulation (Lillehammer et al., 2011).These simulation studies showed the implications and potential of using genomic selection in improving genetic gain of testing sires and dams in nucleus herds. Other researchers tried to evaluate the use the genomic testing technology in commercial dairy farms for applying genotyping to test replacement animals for selection and culling decisions (Weigel et al., 2012; De Vries and Salfer, 2013). The advantage of genomic testing is the ability to accurately identify superior females and males after birth, which consequently helps the farmers make better informed breeding and culling decisions based on the objective of the herd (De Vries and Salfer, 2013). Both studies used static Monte Carlo simulation model to test the percentage of animals selected and the availability of prior information about the animals. Both studies found that the gain from genomic testing of heifers exceeded the cost of test (cost effective), especially when the pedigree and phenotypic information were available for the young animals and small fraction of the young population needed to be tested (Weigel et al., 2012; De Vries and Salfer, 2013).

2.4.4.4. ***Dairy Technology Analysis***

New technology is being introduced to the dairy industry on a regular basis. These technologies could be in genetics (discussed above), reproduction (discussed later), and production. Simulation has always played an important role to explore the cost-benefit and feasibility of a new technology with minimal cost. For example, Hyde and Engel (2002) estimated the breakeven value of a robotic milking system and Bewley (2008) examined the economic feasibility of automated body condition scoring in dairy cattle using static Monte Carlo simulation. Both models used @Risk add-in for Microsoft Excel to build the model. Both models included stochasticity in economic variables and some production characteristics of herd with different sizes to obtain a good estimate of the economic value of using these new techniques.

2.4.4.5. *Reproductive Performance*

As it was discussed, the first important dynamic, stochastic simulation model was modeling the reproduction in dairy cows (Oltenacu et al., 1980). The model used next-event scheduling method to model four reproductive related events on dairy herds (i.e., parturition, ovulation, embryonic loss and replacement). This method schedules all the events that could happen to a cow at day 0 of the simulation and update attributes of the cows whenever an event occurred. For example, after parturition cows should start the ovulation, which followed a lognormal distribution (Oltenacu et al., 1980), given the cow survived the early lactation risk of culling and diseases. This method and the parameters and distributions used in this study was later used by other researchers to simulate dairy herds to study the economic impact of factors related to the reproductive performance on dairy herds (Marsh et al., 1987; Kinsel, 1998; De Vries, 2001). Detail of the next-event scheduling approach will be discussed in Chapter 6.

Many models have been built to study different aspects of reproductive performance and its effect on the farm's profitability. For example, some models have focused on the important factors that have a great impact on overall reproductive performance (e.g., conception rate, estrus detection rate; (Inchaisri et al., 2010)), others on evaluating the economic value of different reproductive protocols and programs (Olynk and Wolf, 2009; Galvão et al., 2013; Rutten et al., 2014), and few studies on the potential implications of dry periods or determining the optimum length of voluntary waiting period (Halasa et al., 2010; Inchaisri et al., 2011).

Inchaisri et al. (2010) analyzed economic value of 3 different reproductive performance scenarios (good, average, and poor), and explored the impacts of reproductive factors (e.g., ovulation rate, estrus detection rate, conception rate, incidence rate of postpartum disorders, voluntary waiting period) on the final economic value. The scenarios were built by changing important factors. For example, the conception rate of a good, average, and poor scenario was 0.7, 0.5, and 0.3, respectively. Using static Monte Carlo simulation built in @Risk Excel add-in they showed that the economic loss due to poor reproductive performance could translate to €231 total loss compared to a good performance. Important economic factors on the reproductive efficiency were involuntary culling cost and revenue from milk production. Variation of conception rate, estrus detection rate, and voluntary waiting period had a large impact on the overall economic value of a given scenario (Inchaisri et al., 2010).

Many different reproductive management programs (e.g., Ovsynch, Cosynch, Double-Ovsynch) have been developed and introduced to the farmers over the years. Given the large numbers of such programs, economic assessments of these programs are beneficial to help decision makers choose economically optimal programs (Olynk and Wolf, 2009). In a study by Olynk and Wolf (2009), herds under Ovsynch and Cosynch reproductive managements were simulated using @Risk Excel add-in. Then, using the stochastic dominance methods the best programs were identified. The study showed that both Ovsynch and Cosynch were preferred over heat detection and farmers, and managers would prefer Ovsynch to Cosynch, regardless of the attitude towards risk (Olynk and Wolf, 2009). In another study using dynamic, stochastic Monte Carlo simulation Galvão et al. (2013) did a comprehensive study on comparing 100% timed artificial insemination, heat detection, and combination of synchronization and heat detection. In this study Presynch-Ovsynch was used for the first insemination and Ovsynch for second and subsequent services. The results were evaluated under 2 levels of important factors on the reproductive performance of herd such as estrus detection rate and its accuracy, compliance of the reproductive protocol, and milk price. In this study a daily model simulated 1,000 cows using Netlogo software and ran until steady-state. The results showed that regardless of the levels the combined program with 60% estrus detection rate always outperformed the 100% timed artificial insemination protocol (Galvão et al., 2013). Further discussion of the actual economic value of different reproductive protocols can be found in Chapter 5.

Heat detection is one of the factors that affect reproductive performance. The average efficiency of visual estrus detection varies between 40 to 55% among studies (Rutten et al., 2014). Activity monitors are automated estrus detection systems that try to improve this efficiency by using sensors and the behavior change in dairy cattle. These systems are expensive and economic feasibility of such systems seems appropriate for large upfront investments. Rutten et al. (2014) used dynamic, stochastic simulation to explore the feasibility of such investment. In that study, a 130-cow herd was simulated under two scenarios of visual heat detection (50% rate) and activity meters (80% sensitivity; 95% specificity). A 11% internal rate of return for investing in activity meters were found in the study and the driving factor was the visual estrus detection rate (Rutten et al., 2014).

2.4.4.6. *Nutrition and Grouping Strategies*

Considering that the field trials in nutrition studies are the norm, the number of simulation models in nutrition is much smaller than other fields. For instance, simulation was used to compare different grouping strategies (Williams and Oltenacu, 1992). In that study a dynamic simulation model was used to compare 7 different strategies to group the cows for feeding and its impact on the income over feed costs (**IOFC**). These strategies were: 1) energy and protein requirements per kg of dry matter intake, 2) required energy and protein per kg of NDF, 3) DIM, 4) test day milk, 5) test day fat corrected milk, 6) kilograms of fat corrected milk per kg of

$BW^{0.75}$ (dairy merit), and 7) dairy merit weighted by DIM. Each of these strategies was tested under 2 or 3 feeding groups after running the model for 10 years. The study showed that in terms of annual IOFC the most effective strategies were those that grouped the cows based on the nutrient concentrations in terms of DMI or NDF (strategies 1 and 2; often called cluster methods (McGilliard et al., 1983)), and the least effective was grouping the cows based on the test day milk (Williams and Oltenacu, 1992). The effectiveness of the cluster method (strategy 1) was also proved in a static simulation study by St-Pierre and Thraen (1999). In that study, the authors used static Monte Carlo simulation to simulate populations of cows fed different NE_L and CP concentrations to find the optimum allocation of NE_L and CP. The results suggested that the optimum allocation depended on the number of feeding groups and the herds doing just one group fed the cows to higher concentration of NE_L and CP, which resulted in nutrient wastage in manure (St-Pierre and Thraen, 1999). Chapter 6 of this thesis describes the development and application of a dynamic, stochastic Monte Carlo simulation to estimate an economic advantage of having multiple feeding groups instead of feeding all the cows one TMR. Further discussion about the potential gain in IOFC obtained by having more than one feeding group is provided in Chapter 6.

2.5. Decision Support Systems (DSS)

2.5.1. Introduction to DSS

In previous sections different modeling techniques that could assist researchers to develop a rather precise model of dairy herd systems were introduced. Results from these models under different conditions could be helpful in understanding dairy herd systems. However, the modeling itself would not be of any assistance to the end-users (e.g., farmers, managers, consultants) unless the results would affect the decision made on dairy herds under their specific conditions. Decision making is a complex multi-step process which starts by gathering intelligence (gathered information from the system to find problems and opportunities), continues by designing valid alternative choices, and finishes by making a choice from a pool of alternatives (Wierzbicki and Lewandowski, 1989; Oz, 1998). Decision support systems (DSS) are the links between the underlying analytical models and the decision making process to assist decision makers to make informed decisions.

2.5.2. DSS Definition

Decision support systems are a subclass of computer based information systems that support technological and managerial decision making by providing useful information regarding ill-structured or semi-structured problems (Sharma et al., 2011). Structured problems are problems in which their answers are reachable by following sequential sets of steps, and the answer for a given set of parameters is always the same. Problems in mathematics and physics are usually structured (e.g., finding a square root, speed of a falling object; (Stair and Reynolds, 1999)). Ill-structured and semi-structured lacks these key properties.

Plotting all information systems on a continuum (Figure 2.10) displays on one side those systems that just can be used as a database and for report generation (answers to structured problems) and on the other side all expert systems that make decisions themselves and learn over time (Sauter, 2010). On this scale DSS would be placed in the mid part of the continuum with a goal to provide decision support for ever-changing problems in the organization (generally business) that often have more than one right answer (unstructured problems). It should be noticed that the DSS main function is to support decisions made by humans (managers and consultants) not to replace them.

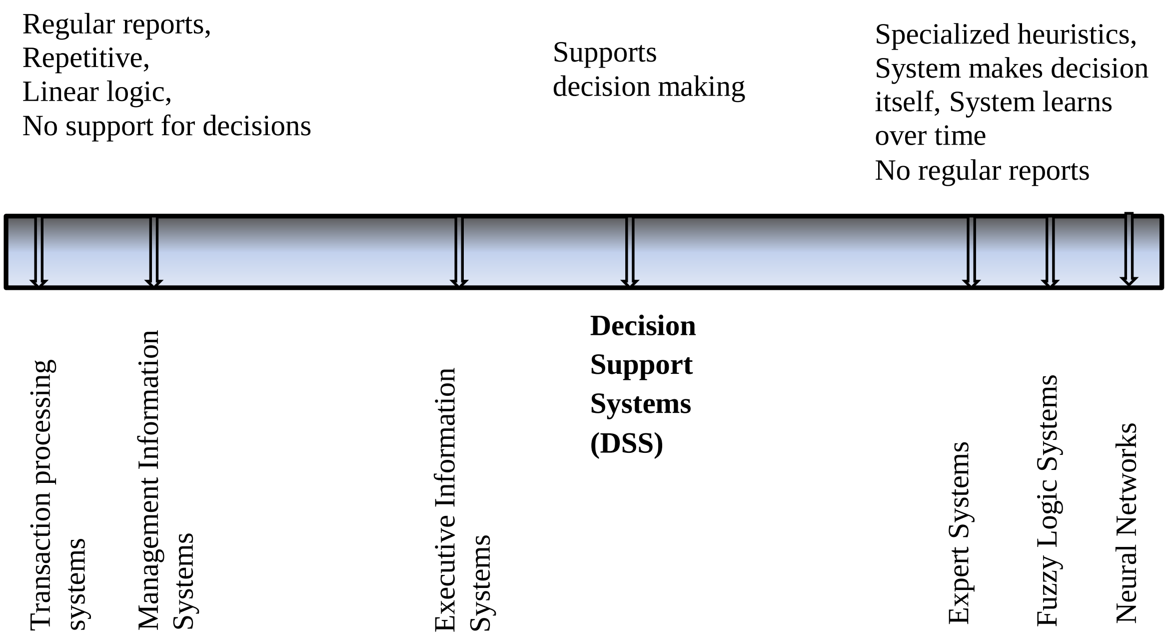


Figure 2.10. Continuum of information systems products (adapted from (Sauter, 2010))

Overall, DSS can be defined as interactive computer programs that use analytical methods, such as regression, simulation, optimization, and decision analysis algorithms, to assist decision makers to analyze the impact of different decisions on the system and select appropriate option based on the gained information (Zwass, 1997; Agrahari and Tripathi, 2012).

2.5.3. DSS Components

Every DSS has three fundamental components as follow (Oz, 1998; Stair and Reynolds, 1999):

1. The database (or knowledge base)
2. The model (based on the context and the goal)
3. The user interface (or dialog management module)

The first module is a database that allows decision makers to conduct the gathering intelligence phase of the decision making process. This module could either be integrated with a database management system or it could obtain the data manually. The model module is based on the overall goal and design phase of the decision making process and tries to turn the data into information for decision makers. All the models described in previous sections have the potential to be used in this module of a DSS. The final component is the user interface and is the part that provides the user the ability to change the parameters and observe the results in a user-friendly environment (Oz, 1998; Stair and Reynolds, 1999).

Although DSS have been classified based on different criteria (Zwass, 1997; Stair and Reynolds, 1999; Agrahari and Tripathi, 2012), the Wierzbicki and Lewandowski (1989) classification, based on practical development of these systems in applications and research, seems the most appropriate for dairy farming applications:

1. Simple tools for managerial decision support (these can be used as building blocks of other main DSS). Examples of these tools are simple access databases and spreadsheet programs.

2. DSS based on logical models and logical inferences. The main function of these systems is to help and identify logical patterns in decision situations. These systems typically include logical programming, expert systems and tools in artificial intelligence.
3. DSS based on analytical models, multi-objective optimization and choice. These models try to find the best choice among alternatives. These systems include a computerized model of the system using simulation with (or without) complex optimization models to evaluate different alternatives.

2.5.4. Development Process of DSS

Developing a complete functional system that does everything from scratch is hardly feasible. Adding depth to the system functionality is the long-term objective and establishing the scope of the system should be the priority in the DSS development process (Bennett, 1983). Thus, the system development has a circular life cycle that starts and ends with planning as it is depicted in Figure 2.11. Planning refers to identifying and selecting the system for development, assessing project feasibility, and creating a development plan. The next step is to analyze business requirements and creating a flow and process diagrams of the decision problem. Design phase works on selecting the model based on understanding the business requirements. The design phase will follow by an implementation which is the development of the model in the computer. Finally, the support or feedback starts which could be used to fine-tune and revise the assumptions or modeling part. Thus, system development follows an incremental, adaptive, iterative design process which helps the system evolve over time (Bennett, 1983; Oz, 1998; Stair and Reynolds, 1999). The same development cycle can be found in management, which is called management cycle (Kristensen et al., 2006).

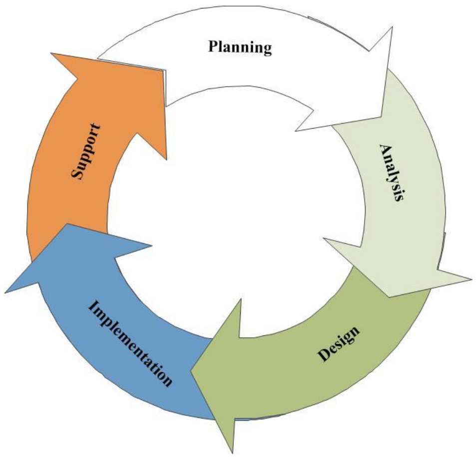


Figure 2.11. The system development life cycle (adapted from (Oz, 1998))

2.5.5. DSS in Dairy Herd Management

Over the years the acceptance of DSS in dairy herd systems has been on the rise and in today's uncertain and risky market using such tools seems to be more appealing than ever. One of the most essential parts of today's dairy farms is the database management system, to "*accurately*" record all the events on a farm (Kristensen et al., 2006). However, as it was discussed and illustrated (Figure 2.10), management information systems are reporting tools with usually no decision support capability. To be valuable to farmers, data needs to be converted to information required in the context of a problem at hand. DSS are the means for creating valuable information from large amounts of data that otherwise would not be that informative. Currently, one could find a decision support tool to assist them in getting information from different aspects of dairy herd management. For example, Cabrera, (2012a) provided a suite of more than 30 DSS with a goal of assisting dairy farmers and consultants in their complex daily decision making, available at (http://WWW.DairyMGT.info). Another great source, but more limited in terms of a range of tools, for DSS is developed by Galligan (Professor of Animal

Health Economics, University of Pennsylvania School of Veterinary Medicine; http://dgalligan.com/).

All the modeling techniques that have been described in this thesis have the capability to become DSS. However, except few research studies in the literature, many of the described models stayed at the research part of modeling and did not evolve into a user-friendly DSS. The following section provides some DSS descriptions that originated from scientific literature and are applied to dairy herd management.

2.5.6. Nutrition Management and Feeding Systems

Feed cost being one of the major variable costs on dairy farms makes this group of DSS valuable tools to dairy farmers. In addition, environmental concerns regarding dairy production systems excretion (e.g., C, N, P) into the environment and potential government policies in the future drive decision makers to appropriate tools to better manage their dairy herd with respect to feed the cows more precisely.

There are a number of tools available for managing feeding costs that could evaluate the IOFC for a specific lactation, feed prices, and is responsive to different feeding strategies throughout the lactation (Cabrera, 2012a). The other example from this category is the “FeedVal v6.0” for ranking the feed costs available in the market based on different nutrients of the given feed. For the issue of environmental stewardship a whole farm simulation and optimization model developed by Cabrera et al. (2006), which could be used to balance between the nutrient concentration in the diet and the requirements with respect to the potential N leaching. The model translated to a DSS (Dynamic Dairy Farm Model) is also available on-line. There number of other tools targeted at the nutrition and risk management of feed which are also available on-line (Cabrera, 2012a).

2.5.7. Reproductive Management and Culling Decisions

In terms of DSS support and tools optimal culling, insemination decisions, and reproductive management are major aspects of a dairy farm that have gained extra attention. This could be due to their high impact on the farm's profitability (van Arendonk, 1985b). Major complexity and possible interactions between different biological and managerial factors on dairy farms, and increasing number and complexity of available reproductive programs in the market (Giordano, 2012) makes DSS well-suited for these type of unstructured problems. However, due to the complexity involved in culling and reproductive management the modeling has usually stayed at the research level. Following are some of the models that evolved into user-friendly tools for end users.

One of the first tools available was for making optimal culling decisions based on the marginal net return calculations (Groenendaal et al., 2004). The simplicity of this model compared to the DP model made it possible to make a user-friendly spreadsheet model. The tool (EconCow or OptiCow; not found online at the time of this work) was envisioned to run fast and evaluate the cows based on their expected value. During the same year De Vries (2004) published a study for finding optimal replacement decisions in dairy cows using the DP model. A Few years later (2007) the resulting tool (DairyVIP1.1) was released to assist farmers, and currently (2015) the latest release is available (DairyVIP2.1; http://dairy.ifas.ufl.edu/tools/). The DairyVIP2.1 uses Microsoft Excel as the user interface, and all the complication of the DP model is hidden in C++ back-end codes for high performance. This is one of those tools that has been used by other researchers (Olynk and Wolf, 2009; Inchaisri et al., 2011, 2012; Galvão et al., 2013). Cabrera (2012b) developed a simple formulation of the sub-optimal replacement problems in dairy cows using Markov chain simulation, which was translated into an easy to use spreadsheet program for economic evaluation of individual cows on the farm. The tool was later transferred online and has the capability to also evaluate the value of all the cows from an input dairy herd (http://dairymg.info/tools/cow_value_resp/). In a recent study the pattern and behavior of DP optimal decisions and corresponding expected value of the cows (RPO) was successfully learned by a machine learning algorithm to make the DP based optimal replacement decision faster and applicable to be used online DSS (http://dairymgt.info/tools/rpo_calc/ (Shahinfar et al., 2014)).

Many studies have also modeled the reproductive management of dairy herds (Olynk and Wolf, 2009; Inchaisri et al., 2010; Galvão et al., 2013). However, one of the first DSS for assessing the economic and reproductive performance of different reproductive protocols was developed by Giordano et al. (2011). This spreadsheet tool combined Markov chain simulation model to estimate the herd structure with multiple partial budgets to calculate the net present value of different reproductive programs (Giordano et al., 2011b). Later the spreadsheet model was named ReproMoney$ and was made available for end-users with an easy to use interface. Different versions of this DSS were released until the time that another DSS using daily Markov chain model was developed based on Giordano et al. (2012). This latest DSS has the ability to compare economic and reproductive performance of two reproductive protocols, giving the decision makers a better idea about the expected performance of a given change in the reproductive performance. More detail about this tool is covered in Chapter 7.

2.5.8. Other DSS

Over the years the original SimHerd model developed by Sorensen et al. (1992) has been the core of overall 28 papers, and over 10 theses studies (Ettema and Østergaard, 2015). SimHerd is a mechanistic, dynamic, and stochastic weekly simulation of dairy herds. It was evolved over the years and different releases of this DSS had been used in many scientific papers (Sorensen, 1998). The evolution and subset of studies is summarized in Table 2.3. Currently, the web version of the SimHerd has been made available as a DSS tool for health economic analysis, generating added value for dairy farmers in Denmark, and great opportunity for researchers in different fields to test different hypothesis and better understanding of dairy systems (Ettema and Østergaard, 2015).

The integrated farm system model (IFSM) is a whole-farm simulation model that links the dairy herd with all other processes on the farm such as machinery, crop production (Rotz et al., 2011). This model is detailed in simulating the environmental impacts of a dairy (and beef) production system, including gas emissions, nitrate leaching, phosphorous runoff, and carbon footprint assessment of production systems. The model can also be used to determine production costs, income of each year by including the seasonality in the calculations. The model has been used mostly in whole-farm research in the scientific literature (Rotz et al., 2011).

There are many other tools available online that could be used to plan ahead and make better informed on-farm decisions to achieve a better performance. For these tools one can review a book chapter by Cabrera (2012a) on available DSSs in the dairy industry.

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Chapter 3

Application of Dynamic Programming and Markov Chain to Evaluate the Herd Value

The effect of reproductive performance on the dairy cattle herd value assessed by integrating a daily dynamic programming model with a daily Markov chain model

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3.1. ABSTRACT

The objective of this study was to determine the effect of reproductive performance on dairy cattle herd value. Herd value was defined as the herd's average retention payoff (RPO). Individual cow RPO is the expected profit from keeping the cow compared with immediate replacement. First, a daily dynamic programming model was developed to calculate the RPO of all cow states in a herd. Second, a daily Markov chain model was applied to estimate the herd demographics. Finally, the herd value was calculated by aggregating the RPO of all cows in the herd. Cow states were described by 5 milk yield classes (76, 88, 100, 112, and 124% with respect to the average), 9 lactations, 750 d in milk, and 282 d in pregnancy. Five different reproductive programs were studied (RP1 to RP5). Reproductive program 1 used 100% timed artificial insemination (TAI; 42% conception rate for first TAI and 30% for second and later services) and the other programs combined TAI with estrus detection. The proportion of cows receiving artificial insemination after estrus detection ranged from 30 to 80%, and conception rate ranged from 25 to 35%. These 5 reproductive programs were categorized according to their 21-d pregnancy rate (21-d PR), which is an indication of the rate that eligible cows become pregnant every 21 d. The 21-d PR was 17% for RP1, 14% for RP2, 16% for RP3, 18% for RP4, and 20% for RP5. Results showed a positive relationship between 21-d PR and herd value. The most extreme herd value difference between 2 reproductive programs was $77/cow per yr for average milk yield (RP5 – RP2), $13/cow per yr for lowest milk yield (RP5 – RP1), and $160/cow per yr

for highest milk yield (RP5 – RP2). Reproductive programs were ranked based on their calculated herd value. With the exception of the best reproductive program (RP5), all other programs showed some level of ranking change according to milk yield. The most dramatic ranking change was observed in RP1, which moved from being the worst ranked for lowest milk yield to the second ranked for highest milk yield. Within a reproductive program, RPO changed based on the stage of lactation at pregnancy. Cows getting pregnant in the early stage of lactation had higher RPO compared with getting pregnant later in the lactation. However, the RPO at calving was similar for early and late lactation pregnancies.

Key words: retention payoff, replacement decision, optimization, simulation

3.2. INTRODUCTION

Reproductive performance affects dairy herd profitability (Britt, 1985; Meadows et al., 2005; Olynk and Wolf, 2008). The association between reproductive performance and profitability is a result of effects on milk yields, available replacement heifers, and voluntary and involuntary culling rates (Olynk and Wolf, 2008). Some studies that have evaluated the economic impact of different reproductive programs using simulation models (Olynk and Wolf, 2009; Cabrera and Giordano, 2010; Giordano et al., 2011a,b) focused on the impact of reproductive programs alone. However, replacement decisions also greatly affect a herd’s profitability (van Arendonk, 1985b).

Several biological and economic factors should be considered in order to make optimal replacement decisions. The most important factors are milk production, pregnancy, stage of lactation, and the value of a replacement heifer. Dynamic programming (**DP**), also known as Markov decision process (**MDP**), is an optimization technique that can handle all of these factors. Dynamic programming models have been developed to optimize culling decisions in dairy herds over the past several decades (Smith, 1973; van Arendonk, 1985b; Kristensen, 1988; De Vries, 2004). The stage length of the DP models in these studies varied from 1 yr to 1 mo. However, farmers are making these decisions on a daily basis. Thus, the applicability of these models for practical farm decision making has been limited (Nielsen et al., 2010).

Recently, Nielsen et al. (2010) used a daily stage length on DP to find the optimal replacement policy in dairy herds. They used a hierarchical MDP algorithm, developed by Kristensen (1988). Emphasis in that study was given to building a DP model to use daily milk yield performances based on the modern milking system in Denmark (Nielsen et al., 2010). Nonetheless, the Nielsen et al. (2010) study did not address the effect of different reproductive programs on replacement decisions and retention payoff (**RPO**, the expected profit from keeping the cow compared with immediate replacement). Hence, the need still exists to evaluate the combined effect of reproductive performance and optimal replacement decisions. A Markov chain simulation model could be a useful technique to approximate the herd structure (De Vries, 2004) used to calculate the weighted average RPO after determining the optimal decisions for all cow states with a DP model.

The main goal of this study was thus to assess the economic impact of reproductive performance under optimal replacement policies. The specific objectives were (1) to develop a daily DP model to compare different reproductive programs' herd values by using the herd structure found with a daily Markov chain model, and (2) to show the effect of pregnancy time on the RPO within specific reproductive programs.

3.3. MATERIALS AND METHODS

A value iteration method (De Vries, 2004) was used in this study to find the optimum replacement policies of the DP problem. After optimizing replacement decisions, a Markov chain model was applied to simulate the herd structure over time (De Vries, 2004; Cabrera, 2010; Kalantari et al., 2010). A daily Markov chain model developed by Giordano et al. (2012) was used to correspond to the dimensions of a daily DP model. Finally, a herd value was calculated by multiplying the RPO resulting from the DP model by the proportion of cows from the Markov chain results. This herd value thus implicitly captures all optimal replacement decisions.

3.3.1. DP Model

Four state variables were included to describe cows in the DP model. Cow state was defined by milk class (c = 1 to 5), lactation number (l = 1 to 9), DIM (d = 1 to 750), and days in pregnancy (DIP; p = 0 to 282). Multiplying all these dimensions creates over 9 million total cow states, but not all of these states are possible because of biological or imposed constraints. Biological constraints indicate that DIP is always greater than DIM. Imposed constraints determine a voluntary waiting period (**VWP**) and last DIM for breeding services. After excluding impossible states, the total number of possible states in the calculations was >3 million. For each state variable, several stochastic elements or transition probabilities are included in the model, such as the probability of abortion, pregnancy, or involuntary culling, and the probability of transition to different milk classes. All of these transition probabilities were accordingly defined on a daily basis.

The daily DP model presented here was developed following the monthly model developed by De Vries (2004). However, in the current daily model, the stage variable was deleted from the dimensions of some of the equations. The reason for doing this was to avoid the out-of-memory exception error in the Windows platform due to the large dimension of the model with daily stages. With this modification, the very large problem was built as a backward induction (value iteration) method without affecting the results. Other modification to the De Vries (2004) formulation was the inclusion of the transition probabilities of abortion (De Vries, 2006). The expected net present value (**NPV**) of the cash flow ($F_{c,l,d,p}$) under an optimum policy was

$$F_{c,l,d,p} = Max\left[Keep_{c,l,d,p}, Repl_{c,l,d,p}\right], \qquad [1]$$

Where $Keep_{c,l,d,p}$ = expected NPV of keeping the cow given the optimal decisions in the remainder stages and $Repl_{c,l,d,p}$ = expected NPV of replacing the cow given the optimal decisions in the remainder stages:

$$Repl_{c,l,d,p} = SELL_{l,d} + FH_t, \qquad [2]$$

$$FH_t = -C + \delta \sum_{c=1}^{5} PH(c)\left\{REV_{c,1,1,0} + \left[1 - Pinv_{1,1}\right] \times F_{c,1,2,0} + Pinv_{1,1} \times \left[SELL_{1,2} + FH_{t+1}\right]\right\}, \qquad [3]$$

where $SELL_{l,d}$ = carcass value; FH_t expected NPV of cash flow for a replacement heifer entered at stage t; C = cost of replacement heifer; PH (c) = probability of replacement heifer with production level c; $REV_{c,l,d,p}$ = net revenue for current state; $Pinv_{l,d}$ = probability of involuntary culling at each day and δ = discount factor.

The expected NPV for the keep decision depends on the state of the cow. Following are the keep value calculations formulas:

1. If the cow was eligible for insemination (p = 0 and $VWP \leq d \leq 300$) then the keep value depended on the insemination cost (PregCost):

$$Keep_{c,l,d,0} = REV_{c,l,d,0} + PregCost_d + \left(\left(\left(1 - Pinv_{l,d}\right) \times Ppreg_d \times \sum_{m=1}^{5} Pmlk_{c,m} \times F_{m,l,d+1,1} + \left(1 - Pinv_{l,d}\right) \times (1 - Ppreg_d) \times \sum_{m=1}^{5} Pmlk_{c,m} \times F_{m,l,d+1,0}\right) + Pinv_{l,d} \times \left(SELL_{l,d+1} + FH_{t+1}\right)\right) \times \delta, \quad [4]$$

Where $Ppreg_d$ = daily probability of pregnancy and $Pmlk_{c,m}$ = daily probability of changing among milk classes.

2. If the cow was open in their last possible DIM (p = 0 and d = 750) or if the cow was in the last day in pregnancy and last possible lactation (p = 282 and l = 9):

$$Keep_{c,l,d,p} = REV_{c,l,d,p} + \left(SELL_{l,d+1} + FH_{t+1}\right) \times \delta, \quad [5]$$

3. If the cow was open and not eligible for insemination (p = 0 and $d < VWP$ or $300 \leq d \leq 749$) or if the cow was pregnant ($1 \leq p \leq 281$):

$$Keep_{c,l,d,p} = REV_{c,l,d,p} + Pregchk_p + \left(\left(\left(1 - Pinv_{l,d}\right) \times \left(1 - PAbor_p\right) \times \sum_{m=1}^{5} Pmlk_{c,m} \times F_{m,l,d+1,p} + \left(1 - Pinv_{l,d}\right) \times ProbAbor_p \times \sum_{m=1}^{5} Pmlk_{c,m} \times F_{m,l,d+1,0}\right) + Pinv_{l,d} \times \left(SELL_{l,d+1} + FH_{t+1}\right)\right) \times \delta, \quad [6]$$

Where $Pregchk_p$ = cost of pregnancy diagnosis and $ProbAbor_p$ = probability of abortion. The abortion term in equation [6] was excluded for cows with less than 30 DIP. The keep value of entering a cow to a new lactation was calculated by adding a constant calf value to equation [6].

4. Finally, if the cow was calving (p = 282), but was not in the last lactation ($l < 9$):

$$Keep_{c,l,d,282} = REV_{c,l,d,282} + \left(\left(\left(1 - Pinv_{l,d}\right) \times \sum_{m=1}^{5} Pmlk_{c,m} \times F_{m,l+1,1,0}\right) + Pinv_{l,d} \times \left(SELL_{l,d+1} + FH_{t+1}\right)\right) \times \delta, \quad [7]$$

The RPO for the current stage can be calculated based on the keep and replace values using the following equation:

$$RPO_{c,l,d,p} = Keep_{c,l,d,p} - Repl_{c,l,d,p}, \qquad [8]$$

The RPO can be either positive when the keep value is more than the replace value or negative when the replace value is higher than the keep value. The RPO can be used to rank cows for replacement decisions in a herd. Higher RPO represents a more valuable cow and RPO below zero means culling of the cow is preferred.

3.3.2. Markov Chain Model

A daily Markov chain model[1] (Giordano et al., 2012) was used to simulate dairy herd dynamics under studied reproductive performances. The Markov chain model found the herd structure at steady state or proportion of cows at each defined state (De Vries, 2004; Cabrera, 2010; Kalantari et al., 2010). The Markov chain model resembled all the daily DP state dimensions, except the milk classes.

3.3.3. Herd Value Calculation

The herd value was defined as the herd's weighted average RPO calculated as follow:

$$\text{Herd Value} = \sum_{l=1}^{9}\sum_{d=1}^{750}\sum_{p=0}^{282}(p_{l,d,p} \times RPO_{l,d,p}), \qquad [9]$$

Where $p_{l,d,p}$ = proportion of cows in each state at steady state from the Markov chain model and $RPO_{c,l,d,p}$ = RPO for each state determined by the DP model. This equation calculates the weighted average of RPO according to the proportion of available cows at each state. The herd

[1] For simple referral a brief explanation of the model is provided in Appendix 2

value was calculated separately for each one of the 5 different milk classes and for each one of the defined reproductive programs.

3.3.4. Computer Implementation

The model was developed as a standalone executable program with Visual Basic.Net 2010 (Microsoft Corp., Seattle). Input variables were in the form of spreadsheet files and results were gathered as comma separated files.

3.3.5. Model Parameters

3.3.5.1. *Milk Production*

The incomplete gamma function (Wood, 1967) was used to estimate milk production throughout lactation. The Levenberg-Marquardt algorithm was used to minimize the difference between milk yield observations and estimated values. Factors of 5, 10, and 15% for milk production depression due to pregnancy were applied at 5, 6, and 7 mo in pregnancy, according to De Vries (2004). These monthly probabilities were converted to daily probabilities.

3.3.5.2. *Milk Class Transition*

The 15 milk classes as described by van Arendonk (1985) were merged to 5 milk classes. The fitted data were set to be an average milk class (third class) and other classes were set as a factor to this average milk class. For example, the lowest milk class was set to 76% of the third milk class and the highest milk class produced 124% of the third class. In van Arendonk (1985) study, the repeatability of milk production was set to 0.55 monthly. Corresponding repeatability was empirically found to be 0.99 on a daily basis.

3.3.5.3. *Carcass Value and Feed Intake*

The daily BW for each state was calculated by the Korver function (Korver et al., 1985) as described by van Arendonk (1985). The model was parameterized to replicate the BW changes throughout lactation as shown in the NRC (2001) BW function. Body weight was also used to calculate the daily DMI. Dry matter intake was calculated according to NRC (2001) as a function of BW and 4% FCM.

3.3.5.4. *Involuntary Culling and Reproductive Performance*

The daily probability of involuntary culling for every cow state defined in the model was calculated based on the monthly involuntary culling described by De Vries et al. (2010).

A subset of 5 reproductive programs (**RP**) from Giordano et al. (2012) was studied (Table 3.1). Originally, 19 different RP were simulated in Giordano et al. (2012) to encompass plausible ranges of reproductive performance observed in commercial dairy farms. They compared the economic and reproductive performance of a program that used 100% timed AI (**TAI**) for all AI services with combined TAI plus estrus detection programs at different levels of estrus detection (Giordano et al., 2012). In these combined programs, the probability of insemination after estrus detection increased from 30 to 80% with 10-percentage-unit intervals at 3 conception rates of 25, 30, and 35% (Giordano et al., 2012). Therefore, each reproductive program was represented by a vector of daily pregnancy probabilities depending on their levels of estrus detection and conception rates. It has been shown that when estrus detection is added before or between TAI services, a conception rate reduction of those cows inseminated to TAI is observed (Chebel et al., 2004). A possible explanation of this reduction is that cows not detected in estrus and reaching TAI have lower fertility potential (Keskin et al., 2011). Thus, the baseline 40% conception rate after TAI at 30% estrus detection in first service was decreased by 2 percentage units for each 10% increase in the estrus detection. For second and subsequent services, the conception rate was set at 30% when the estrus detection was between 30 and 50% and at 28% when estrus detection was between 60 and 80% (Giordano et al., 2012).

The 5 RP studied here were selected to represent all the range of reproductive performances observed in Giordano et al. (2012): the 100% TAI program and the combined programs at 2-percentage-unit intervals of 21-d pregnancy rates (21-d PR; Ferguson and Galligan, 1999). The first program used 100% TAI for all AI services and the other programs combined TAI with estrus detection with different levels of service and conception rates, (Table 3.1). The first program (RP1) used Presynch-Ovsynch for the first postpartum AI and Ovsynch for second and subsequent AI services with a 72-d VWP and an interbreeding interval of 42 d. Combined programs (RP2 to RP5) used the same synchronization protocol as RP1, except that AI after estrus detection was added between the end of VWP (50 d) and the first TAI at 72 DIM and in between TAI services (Giordano et al., 2012). These 5 RP were categorized according to their 21-d PR. The 21-d PR was 17% for RP1, 14% for RP2, 16% for RP3, 18% for RP4, and 20% for RP5.

Total daily reproductive cost for each program was calculated from the costs of labor for estrus detection and hormone injection and the costs of hormones for synchronization of ovulation and AI (Giordano et al., 2012). Additionally, constant costs related to pregnancy diagnosis, assuming rectal palpation, were applied at 39, 67, and 221 DIP. Pregnant cows had a daily probability of pregnancy loss from 30 DIP to term. These probabilities varied according to the stage of gestation. The probability of pregnancy loss was set at 0% for the first 30 DIP, 12.5% from 30 to 45 DIP, 9.9% from 46 to 180 DIP, and 2% from 181 DIP to term (Giordano et al., 2012).

Table 3.1. Summary of studied reproductive programs

Reproductive Program[1]	First AI (%)			Second and subsequent AI (%)			21d pregnancy rate (%)
	ED before 1st TAI[2]	CR[3] ED before 1st TAI	CR TAI	ED before TAI	CR ED before TAI	CR TAI	
RP1	-	-	42	-	-	30	17
RP2	70	25	32	70	25	28	14
RP3	50	30	36	50	30	30	16
RP4	30	35	40	30	35	30	18
RP5	80	35	30	80	35	28	20

[1] A subset of reproductive programs studied in Giordano et al. (2012). RP1 relied only on timed AI (TAI) for first AI with Presynch-Ovsynch protocol and for second and subsequent AI services with Ovsynch protocol, having a voluntary waiting period of 72 d and an interbreeding interval of 42 d; RP2 to RP5 combined TAI with estrus detection between the end of the voluntary waiting period (50 d) and the first TAI at 72 DIM and during the subsequent re-synchronizations.
[2] Percentage of cows AI after estrus detection before TAI.

[3] Conception rate of cows AI after estrus detection before TAI.

3.3.5.5. *Economic Parameters.*

Calves were assumed to be sold immediately, and the revenue from them was a weighted average price for male and female calves ($100). The yearly veterinary cost for an average first lactation cow was set to $50 and increased by $5 each lactation (Groenendaal et al., 2004). These veterinary costs were assigned based on van Arendonk (1985): 33% to the first month of lactation, 11% to the second and third months in lactation, and the rest to the remainder of the lactation. Other inputs are summarized in Table 3.2.

Table 3.2. Economic parameters[1]

Variable	Value
Price	
Milk	\$0.36 /kg
Calf value	\$100/calf
Carcass value	\$1.16/kg
Replacement Heifer cost	\$1,300
Veterinary cost	\$50
Feed cost (Lactation)	\$0.17/kg
Feed cost (Dry period)	\$0.13/kg
Annual Interest rate	10%

[1] Same values used in Giordano et al. (2012).

3.4. RESULTS AND DISCUSSION

3.4.1. Herd Value Difference Between Reproductive Programs

The herd values for 5 RP across 5 different milk yields are presented in Table 3.3. Overall, results showed a positive relationship between 21-d PR and herd value. At average milk yield, RP were ranked based on their herd values: RP5, RP4, RP1, RP3, and RP2 from highest to lowest. This ranking was consistent with that found with only the daily Markov chain model without milk classes (Giordano et al., 2012). However, Figure 3.1 reveals an interesting interaction between milk yield and RP. Every RP except RP5 showed some level of ranking change with relative milk yield. The RP5 with 20% 21-d PR was the absolute best program and did not show any ranking interaction with relative milk yield. The situation for the other analyzed RP was not stable across different milk yield classes. The most dramatic ranking change was observed for RP1 (100% TAI) with 17% 21-d PR. This program's herd value changed from being the worst program at the lowest milk yield to the second best program at highest milk yield. At the highest milk yield, RP4 (18% 21-d PR) was ranked below RP1 (17% 21-d PR).

The most extreme herd value difference ($/cow per yr) between 2 reproductive programs was $77 (RP5 – RP2) for average milk yield, $13 (RP5 – RP1) for 24% below average milk yield, and $160 (RP5 – RP2) for 24% above average milk yield.

The large effect of relative milk yield on herd value is shown in Table 3.3. The average herd value difference ($/cow per yr) between the lowest and the highest milk production across all RP was $1,541, and varied from $1,434 (RP2) to $1,589 (RP1). Main parameters affecting RPO in DP models have been well studied through sensitivity analysis (van Arendonk and Dijkhuizen, 1985; van Arendonk, 1985b; Cardoso et al., 1999; Kalantari et al., 2010; Demeter et al., 2011). These studies have shown, using a monthly stage length DP, that the most important factors affecting RPO values are milk production, price of replacement heifer, and carcass value. The current study also found that the milk production is a very important factor determining the RPO.

Table 3.3. Herd values (US$) for 5 reproductive programs across 5 relative milk yields

Reproductive Program[1]	Relative milk yield to average lactation curve (%)					
	21 d Pregnancy Rate (%)	76	88	100	112	124
RP1[1]	17	156	374	769	1,224	1,745
RP2[2]	14	159	376	729	1,129	1,593
RP3[2]	16	161	385	763	1,190	1,683
RP4[2]	18	167	395	788	1,234	1,741
RP5[2]	20	169	410	806	1,248	1,753

[1] RP1 relied only on TAI for first AI with Presynch-Ovsynch protocol and for second and subsequent AI services with Ovsynch protocol, having a voluntary waiting period of 72 d and an interbreeding interval of 42 d. RP2 to RP5 combined TAI with estrus detection between the end of voluntary waiting period (50 d) and the first TAI at 72 DIM and during the subsequent re-synchronizations.

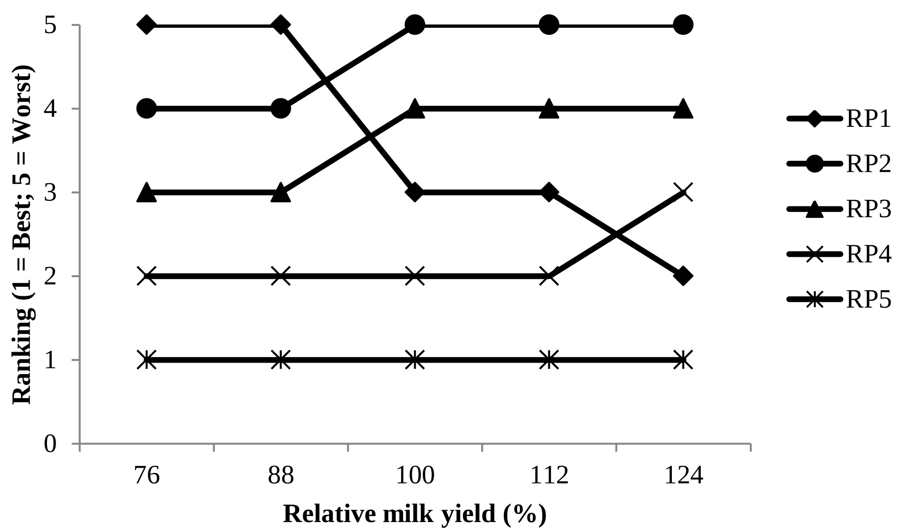

Figure 3.1. Ranking changes of 5 reproductive programs (RP) across 5 relative milk yields (%). Reproductive program 1 relied only on timed AI (TAI) and had a 21-d pregnancy rate of 17%; RP2 to RP5 combined TAI with estrus detection for 21-d pregnancy rates of 14, 16, 18, and 20%, respectively.

The difference in herd values between RP is attributed to the effect of RP on the herd structure (percentage of milking and pregnant versus dry and open cows, and distribution of cows between and within lactations). Figure 3.2 shows the effect of different RP on the herd structure as well as on herd values. Figure 3.2 illustrates the RPO multiplied by the proportion of cows at each state across the first 3 lactations for 3 different RP (RP1, RP2, and RP5). Hence, Figure 3.2 is the daily RPO value weighted by the proportion of cows at each state ($/cow per d). In each lactation, the graph first shows a downward trend, which follows milk production curves. Before parturition, milk production is the main source of cow value, which is reflected by the milk production curve for the average RPO values.

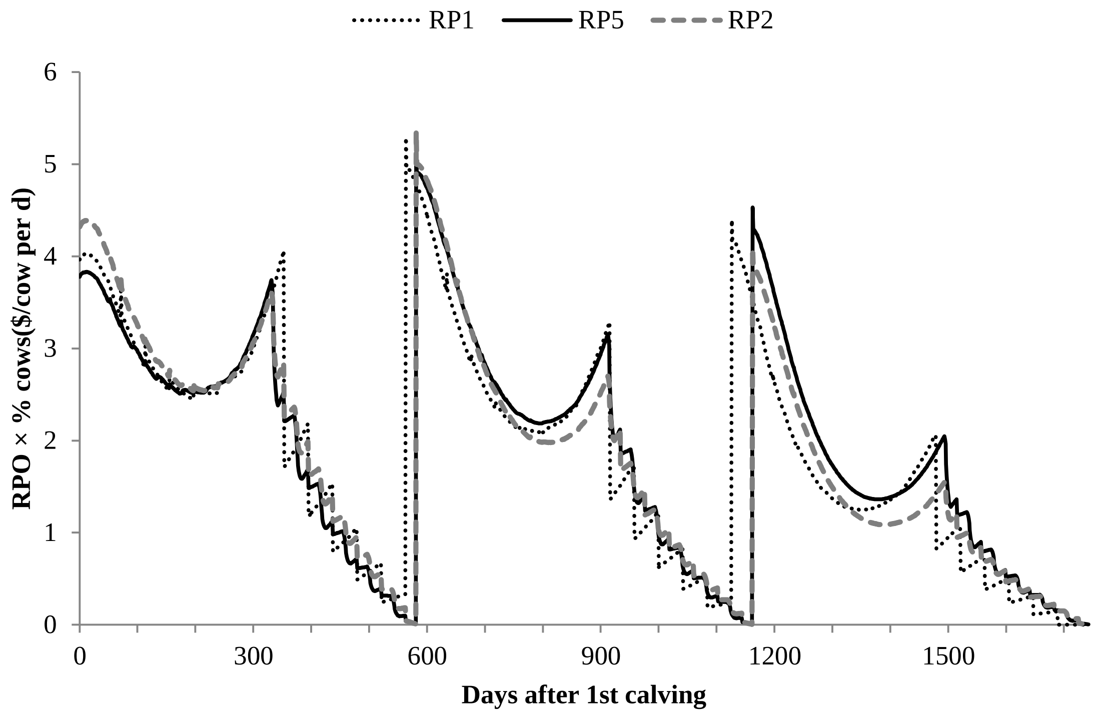

Figure 3.2. Product of retention payoff (RPO) by percentage of cows at each state in the first 3 lactations for 3 reproductive programs (RP). Reproductive program 1 relied only on timed AI (TAI) and had a 21-d pregnancy rate of 17%; RP2 and RP5 combined TAI with estrus detection and had 14 and 20% 21-d pregnancy rates, respectively.

After reaching the nadir, which is a result of the RPO decreasing through lactation and proportion of cows during lactations, RPO shows an upward trend because of the expected value of a newborn calf. Following the first peak during each lactation, a steep, dented downward trend continues, which is a result of 2 factors. The first factor is parturition in a timely manner, based on the reproductive program characteristics. For example, RP1 shows a completely discrete pattern every 42 d, reflecting its TAI interbreeding interval. The other 2 programs show more continuous patterns because of estrus detection breedings occurring in between TAI synchronizations.

The steep downward trend in Figure 3.2 is then the result of decreasing the proportion of cows throughout DIM along with lower RPO late in lactation. It is clear that the RP with lowest 21-d PR (RP2, 14%) has the greatest value in first lactation. This illustrates that RP with lower 21-d PR have more reproductive and non-reproductive culling. Thus, these programs have a higher proportion of cows in first lactation and a lower proportion of cows in second and later lactations, which is less profitable. When comparing RP1 (100% TAI) with RP5, it is clear that after the second lactation, RP1 falls behind the combined program (RP5). This can be mostly attributed to the high percentage of estrus detection (80%) with a high conception rate (35%) in the combined program (Table 3.1). Cows inseminated after estrous detection had a shorter interbreeding interval and greater conception rate than those cows reaching the TAI (Giordano et al., 2012). These differences in conception rate and estrus detection were reflected in a 3-percentage-unit difference in 21-d PR (17 vs. 20%) between RP1 and RP5.

3.4.2. RPO Difference Within an RP

Figure 3.3 shows the effect of DIM and pregnancy time on the RPO. The open cow RPO follows a milk curve pattern. The cow value is the highest at the beginning and gradually decreases through lactation. The unusually greater open RPO on the first day in lactation is a result of adding the value of the newborn calf on this first day. This greater open RPO is obscured in monthly DP models (De Vries, 2006; Kalantari et al., 2010). The pregnant cow RPO curve, which is equal to the difference between the RPO of pregnant and non-pregnant cow at the same DIM, changes based on the time in pregnancy. With increasing DIM at pregnancy, pregnancy value curves (pregnant RPO value) start closer to the RPO of open cows. The pregnant RPO value of a cow becoming pregnant at 200 DIM for second and later lactations was less than the RPO of open cows. For first lactation cows, this occurred at about 260 DIM (data not shown). However, the final RPO of a pregnant cow at parturition was similar in spite of DIM at pregnancy (about $1,450). The lowest RPO value in the daily DP model developed in this study was −$10, which is much less than that found with a monthly model (−$184; Kalantari et al. 2010). A much lower RPO negative value was also found by the daily DP model of Nielsen et al. (2010). Daily RPO is the opportunity cost of keeping a cow for only one more day, and therefore a negative value does not accumulate for longer periods.

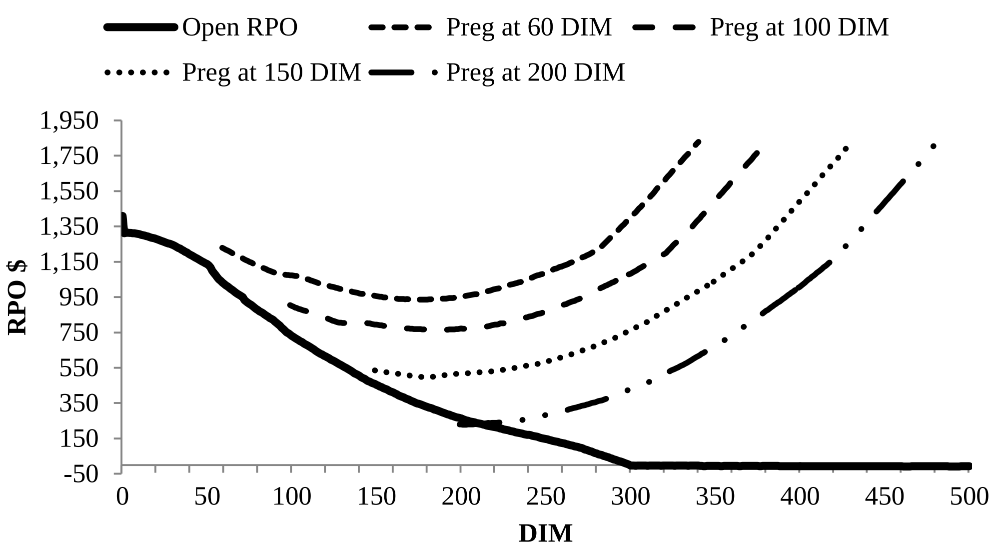

Figure 3.3. Retention payoff (RPO) values for cows at different DIM of pregnancy in the second lactation for the average milk class and reproductive program RP5 (which combined timed AI with estrus detection and had a 21-d pregnancy rate of 20%).

The trends on RPO and the relative values are more important than the actual calculated values (Groenendaal et al., 2004). Therefore, panel A of Figure 3.4 is presented to illustrate the RPO trend throughout 9 lactations for different DIM at pregnancy. It is clear that increasing DIM at pregnancy from 55 to 200 d changes the shape of the curves slightly. The maximum RPO occurred in the fourth lactation and after that, it decreased consistently until the ninth lactation, which was the maximum possible cow life. Similar trends were previously reported in monthly models (Groenendaal et al., 2004; Kalantari et al., 2010). Generally, the RPO of cows decreased after parturition until the point of pregnancy. After pregnancy, the RPO showed a large jump to either a higher or lower value (depending on the DIM at pregnancy; Figure 3.4, panel A). The times of a pregnancy in first and second lactations were obvious: a vertical upward straight line can be seen. However, these lines were obscured after second lactation. During early pregnancy, the RPO usually showed a slight decrease in value. This decrease depended on the lactation and DIM at which the cow was pregnant. The RPO of a cow pregnant at 55 DIM decreased slightly in each lactation and then increased until calving. One difference when pregnancy occurred at 200 DIM was that the RPO increased permanently after pregnancy until the time of calving (panel A in Figure 3.4; Figure 3.3). This difference in trend of the RPO in pregnant cows between early and late pregnancy was mainly a result of the effect of expected milk production (determined by the projected milk production curve) and involuntary culling on the net revenue. That is, cows becoming pregnant late in lactation will have a smaller difference between keep and replace values than cows becoming pregnant early in the lactation. This differential in value is because of events occurring in late lactation: a decrease in natural milk production and increase in involuntary culling, either or both of which may occur. As a result, a higher value is placed on getting cows pregnant late in late lactation. This translates to a permanent increase of RPO of pregnant cows late in lactation (Figure 3.3; Figure 3.4). Despite early or late pregnancy, the RPO of pregnant cows increased to parturition time because of the expected value of the calf for the next lactation (Figure 3.4, panels A and B). To compare the overall value of cows becoming pregnant at different DIM within a defined reproductive program (RP5), the accumulated weighted RPO for each cow state over a cow's lifetime were calculated. Accumulated weighted RPO value for a cow becoming pregnant at 55 DIM was $409 and for a cow becoming pregnant at 200 DIM was $333, a $76 difference between these 2 scenarios. This difference was a result of RPO changes only throughout lactation because the herd structure remained the same for these 2 scenarios. The RPO for cows becoming pregnant later in the lactation was always lower (De Vries, 2004).

The RPO values for the first 3 lactations for RP5, average milk class, and 120 DIM at conception are shown in Figure 4, panel B. This shows the effect of pregnancy loss on RPO at different DIM comparing 2 scenarios of cows having pregnancy loss with one scenario of a cow without a pregnancy loss: one cow has a pregnancy loss at 170 DIM (50 DIP) and is successfully rebred 30 d later; another cow has a pregnancy loss at 220 DIM (100 DIP) and is successfully rebred 30 d later. The RPO of the cows losing a pregnancy decreased dramatically until the next successful conception. This decrease in RPO depended on the DIP at which the pregnancy loss happened. As in the previous case, the effect of pregnancy loss on the overall value was calculated as the weighted average RPO (RPO times the proportion of cows at that specific state) throughout 9 lactations. This value for the cow without a pregnancy loss was $370, $29 greater than the cow with pregnancy loss at 50 DIP (and successful rebred 30 d later) and $36 greater than the cow with pregnancy loss at 100 DIP (and successful rebred 30 d later). It is obvious that pregnancy loss had a considerable effect on the overall RPO of cows in the herd, even though a successful rebreeding occurred soon after.

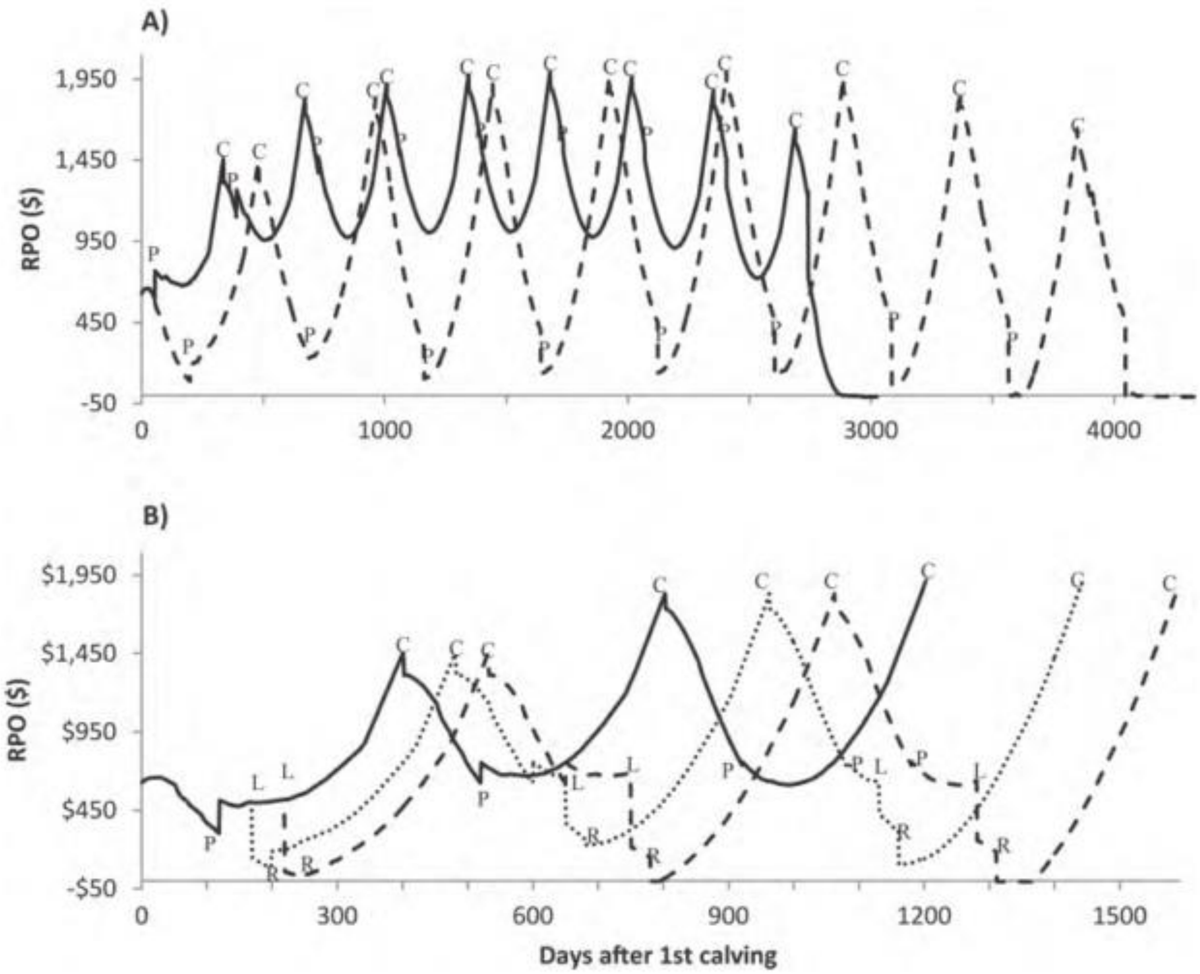

Figure 3.4. Daily retention payoff (RPO) of reproductive program 5 (which combined timed AI with estrus detection with 21-d pregnancy rate of 20%) and average milk class under different scenarios. (A) Pregnancy at 55 DIM (—), pregnancy at 200 DIM (----) during 9 lactations; (B) pregnancy at 120 DIM without pregnancy loss (—), pregnancy at 120 DIM with pregnancy loss at 170 DIM and successfully rebred at 200 DIM (….), and pregnancy at 120 DIM with pregnancy loss at 220 DIM and successfully rebred at 250 DIM (- - - -) during each of the first 3 lactations. Labels show events: pregnancy (P), pregnancy loss (L), successfully rebred (R), and calving (C).

3.4.3. Implication for Dairy Farm Decision-Making and Management

Our results support the notion that an opportunity exists to adjust reproductive programs according to milk classes and therefore according to RPO. The modeling framework could be used for daily decisions of assigning cows to different reproductive management groups based on their RPO. This would promote more cost-effective reproductive programs and therefore overall improved herd value. In addition, results demonstrate an interaction among herd value, relative milk yield, and reproductive programs at different levels of estrus detection and TAI. Therefore, daily reproductive decisions might include, for example, whether to breed a heat-detected cow or not. Cows with higher relative milk yield would benefit more from TAI reproductive programs, whereas cows with lower relative milk yield would be better off with estrus detection programs (Figure 3.1).

Replacement decisions are being made daily on dairy farms and these decisions have a great effect on the herd profitability. Despite their great impact on profitability, these decisions are still made arbitrarily. The most important aspect of the DP model is its ability to rank cows in the herd based on their value. This ranking could be an important guideline to replace the least profitable animals in the herd. Dairy farmers, extension professionals, and farm advisers could take advantage of this information to help decision-making and management on dairy farms. Indeed, the models presented here could be incorporated into software already being used on dairy farms. The current DP model's daily stage length has the potential to provide more accurate RPO and to be used as a guideline in these crucial daily decisions. Moreover, the ranking could be used in conjunction with reproductive decisions, such as distinguishing the quality and type of semen according to daily RPO ranking. A possible refinement of the DP model could include the decision whether or not to breed eligible cows on a daily basis. The DP model could be modified to address a greater range of dairy farms such as organic or grazing farms. For example, those farms having a goal to promote the longevity of milk cows could use suboptimal decisions: this could be addressed by multiplying the keep value in the objective function (Eq. [1]) by a constant factor (α). Our default formulation assumes an α value of 1.

Farms willing to keep cows for longer than their economically optimal lifetime would use an empirical valuation of this factor >1. On the opposite end of the spectrum, farms wanting a faster turnover of animals because of potential herd genetic gain would have this value be <1. The decisions with this modification could be, to some extent, subjective, but could accommodate particular farm objectives.

An important advantage of using daily stage in the models presented in this study is the ability to better represent farm-specific and detailed reproductive management strategies. For example, the models could be adjusted to farms that use natural service by modifying the probability of pregnancy calculations. Because the actual time of insemination is not known, stochastic distribution could be used to simulate possible observed patterns. Therefore, distributions could be applied to simulate the unknown service times. Currently, a similar methodology is implemented to simulate the distribution of cows showing estrus. Although DP optimal decisions are not sensitive to the probability of pregnancy (van Arendonk and Dijkhuizen, 1985), the daily Markov chain model is highly dependent on these probabilities. Additionally, the daily Markov chain model could be used to analyze the cost effectiveness of different pregnancy diagnosis techniques and their interaction with reproductive programs.

Another application of the DP model could be in veterinary treatment decisions (Demeter et al., 2011). Positive RPO represents the expected value of keeping a cow one more day, and negative RPO is the opportunity cost of keeping this cow one more day. A positive RPO could be interpreted as the maximum acceptable treatment cost that could be spent in treating a cow (van Arendonk, 1988). On the other hand, a negative RPO shows the amount of money the farm is losing by keeping the cow one more day. Indeed, the daily DP model could be used by researchers to estimate more accurately the costs of days open, new pregnancy values, or the value of pregnancy losses. Ultimately, both models (daily DP and daily Markov chain) could be integrated with existing models to perform whole-farm studies.

As with any simulation and optimization model, the current models have some limitations. The models presented here do not include health problems, such as mastitis and lameness, or seasonality, which have been shown to have considerable effect on optimal decisions, herd structure, and herd net return (Houben et al., 1994; De Vries, 2004; Cha et al., 2010). These limitations could be overcome by including more state variables. However, inclusion of new states makes the state space of the models grow exponentially. Large DP models could become unsolvable. Hence, a tradeoff exists between including more state variables and decreasing stage length.

3.5. CONCLUSIONS

A daily DP model was developed to evaluate the effect of different reproductive programs on herd value when coupled with a daily Markov chain model. Results showed that herd values were largely influenced by reproductive programs. In addition, an interaction was observed among the herd value, milk yields, and reproductive programs. Results support the notion that reproductive programs or specific reproductive events could be designed according to the individual cow expected production level for improved herd value. Within the same reproductive program, the RPO changed based on the stage of lactation at pregnancy. Cows becoming pregnant early in lactation had greater RPO than cows becoming pregnant later in lactation.

3.6. ACKNOWLEDGMENTS

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Chapter 4

Dairy Cattle Replacement Strategies Optimization vs. Simulation

A comparison analysis of two alternative dairy cattle replacement strategies: Optimization versus Simulation models.

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4.1. ABSTRACT

The objective of this study was to compare the optimal replacement decisions using two alternative state-of-the-art models: the optimization dynamic programming model and the Markov chain simulation model. Lactation, month in milk and pregnancy status were used to describe cow states in a herd in both models. Both models were fed with the same parameters and transition probabilities to make the fairest comparison possible. The cow value calculated by the Markov chain model was compared against the retention pay-off estimated by the dynamic programming model. These values were used to rank all the animals in the herd. Then, the rank correlation (Spearman's correlation) was calculated between results of both models. The overall correlation was 95%, which showed a strong linear relationship between rankings of animals from the two models. Moreover, the lowest 10% ranking cows -which are the most likely replacement candidates- displayed a greater correlation, 98%. Thus, the final replacement decisions with both models were similar. A post optimality analysis was used to explore the effect of the optimal replacement decisions on the herd dynamics and herd net return. The results showed a comparable herd structure by both models. A net return was improved $6/cow per year by using replacement decisions of both dynamic programming model and the Markov chain cow value model.

Key words: herd economics, optimization, replacement policy, simulation

4.2. INTRODUCTION

The ability of farmers to make right decisions at the right times significantly determines the success of any enterprise. This success can be stated as maximizing profit. It has been shown that total profit is highly affected by replacement decisions (van Arendonk, 1984) and reproductive performance (Britt, 1985). Reproductive performance attained special attention in research literature (Olynk and Wolf, 2009; Cabrera and Giordano, 2010; Giordano et al., 2011b, 2012) as a result of its prominent economic impact on the profitability of dairy farms.

Over the past decades several studies have analyzed the optimum replacement interval in dairy herds and factors that affect these decisions (Smith, 1973; van Arendonk, 1985b; Kristensen, 1988; De Vries, 2004; Groenendaal et al., 2004; Demeter et al., 2011; Cabrera, 2012b). Simultaneous accounting of several biological and economic parameters is necessary to determine the optimum time of replacing a cow. Milk production level, pregnancy, stage of lactation, parity and transition probabilities such as involuntary culling, pregnancy, and abortion are considered the most important factors affecting replacement decisions (Kalantari et al., 2010). Approaches that have been proposed to handle these factors and find the optimum replacement strategy including marginal net revenue (**MNR**) (van Arendonk, 1984; Groenendaal et al., 2004), dynamic programming (**DP**) (Smith, 1973; van Arendonk, 1985b; De Vries, 2004), and stochastic simulation models (Marsh et al., 1987; Dijkhuizen and Stelwagen, 1988; Kristensen and Thysen, 1991). The first two methods are based on the production function approach in which the cow's revenue and costs are modeled during cow's lifetime (Groenendaal et al., 2004). The limitation of MNR is its inability to include the variation in expected milk production of the present cow and subsequent replacement heifers, and the genetic gain of replacement heifers (Groenendaal et al., 2004). The DP technique overcomes both of these limitations. However, because its complexity, the usage of DP models has been restricted to research analysis and not for building decision support systems for practical decision-making and farm management. The Monte Carlo stochastic simulation approach has been used to calculate the total expected net returns during next year and that value was used for ranking animals. Kristensen and Thysen (1991) compared the decisions being made by DP and stochastic simulation and reported insignificant difference between the two models.

Recently, Cabrera (2012) used a Markov chain simulation model to find a suboptimal replacement policy. In brief, this method calculates the net present value for a cow and its potential replacement, which could be used to decide whether to keep or replace a dairy cow. This method does not have the complexity of DP models and overcomes the limitation of MNR method because it can include expected variations in the cow and replacement performances. He reported that trend and replacement strategies found with the newly Markov chain model would be similar to those found with DP models. However, such study did not include a formal comparison with a DP model. Consequently, the objectives of this study are to compare the replacement decision strategies reached with a DP and a Markov chain model; and to compare the effect of optimal replacement strategy on the herd structure and net revenue.

4.3. MATERIALS AND METHODS

In this study we compare the outcomes of two alternative models currently used in the literature to offer dairy cattle replacement policies. The DP model was adapted from Kalantari et al. (2010) and the Markov chain model from Cabrera (2012). Both models were set to follow similar specifications and parameters.

4.3.1. Modeling Specifications

Three state variables were used to describe cows in both models. Cow states were defined by lactation number (l = 1 to 10), month in milk (m = 1 to 20), and month in pregnancy (p = 0 to 9; 0 for open cow and 1 to 9 for pregnant). After discounting impossible states, each model had 1,000 possible states. There were also a number of common stochastic elements for transition probabilities such as the probability of abortion, pregnancy, and involuntary culling. These transition probabilities were used to define the flow process of cows among states from one month to another. For example, an open cow could become pregnant in the current month or be involuntary culled (retired because the cow can no longer produce) in next month according to these probabilities (Cabrera, 2012b).

Although both models rely on Markov chains as their underline structure, they have different control mechanisms. The transition probability matrix is the only governing rule that changes states from one stage to another in a Markov chain model. However, there is an extra step at each stage on the DP model, which is to select the optimal action in the current stage for the specified state variables. In other words, the addition of a system control mechanism, which can be defined with the term Markov decision process instead of Markov chain (Gosavi, 2003).

4.3.2. Dynamic Programming Model

The DP model used the ‘divide and conquer’ algorithm to break the multi-stage problem into a series of independent single-stage problems. The objective function was to maximize the net present value of revenues from the current cow and its potential replacements (Kalantari et al., 2010). The objective function can be shown in terms of mathematical notion as follows:

$$F_{l,m,p} = Max\left[Keep_{l,m,p}, Repl_{l,m,p}\right], \qquad [1]$$

Where $Keep_{l,m,p}$ = expected net present value **(NPV)** of keeping the cow in lactation l, month in milk m, and pregnancy p, given the optimal decisions in the remainder stages and $Repl_{l,m,p}$ = expected net present value of replacing the cow given the optimal decisions in the remainder stages. The detailed formulation of calculating the keep and replace values for different states can be found in (Kalantari et al., 2010). Retention pay-off (**RPO**), which is the expected profit from keeping the cow compared with immediate replacement (De Vries, 2004), was calculated using the following equation:

$$RPO_{l,m,p} = Keep_{l,m,p} - Repl_{l,m,p}, \qquad [2]$$

The RPO represents the value of a given cow (represented by *l,m,p*). The RPO can take positive, zero, or negative values. A positive RPO determines that keeping the cow for another month has a higher net return than replacing it, whereas negative RPO means that immediate replacement has a higher net return than keeping the cow. The RPO can be used to rank all cows in the herd to find out the cows that are most likely replacement candidates.

4.3.3. Markov Chain Cow Value Model

A Markov chain model with monthly stage was developed to predict the herd structure at each stage following (Cabrera, 2012b). The NPV of the cow and its replacement is calculated at each stage until the model reaches the condition of 'steady state'. Steady state is achieved when the proportion of cows in all states remain constant in two subsequent stages. Steady state in the model defined here always occurred before iteration number 150th (which is the same as 150 months in the future). Formulas for calculating the proportion of cows at each stage are described in detail in (Cabrera, 2012b).

The NPV of the cow and its replacement were calculated by adding all economic values at each stage from the start of simulation until a time when the model was at steady state. Economic values at each stage were calculated as the sum product of the net revenue of each state and the corresponding herd structure. The formula, following notations in Cabrera (2012), for this calculation follows:

$$NPV = \sum_{i=1}^{150} \partial \left[\sum_{l=1}^{10} \sum_{m=1}^{25} \sum_{p=0}^{9} (Mi - Fc + Ci - NRCc - Mc - RCc - Rc)_{l,m,p} \times (COW)_{l,m,p} \right]_i \quad [3]$$

Where ∂ is interest rate, Mi milk income, Fc feed cost, Ci calf income, NRCc non-reproductive culling cost, Mc Mortality cost, RCc reproductive culling cost, Rc reproductive cost, and COW the proportion of cows (herd structure) at each stage (i) for given state variable (represented by *l,m,p*). After finding the NPV for both the cow and its replacement the cow value was estimated by using the following equation:

Cow Value = NPV Cow – NPV Replacement – (Replacement Cost – Salvage Value – Calf Value)

[4]

This cow value could then be used for deciding whether to keep or replace a cow based upon the sign of the value. Positive cow value (like positive RPO) means that the cow would bring more net revenue than its replacement and therefore the best decision would be to keep the cow. A negative cow value means that replacement is more profitable than keeping it.

4.3.4. Shared Models Parameters

4.3.4.1. *Milk Production*

The MilkBot function (Ehrlich, 2011) was used to fit milk production curves for the first, second and third and later lactations. The MilkBot predicts milk yields, Y(m), as a function of time after parturition or months in milk, m. Four parameters, a (scale), b (ramp), c (offset), and d (decay), control the shape of the lactation curves (Ehrlich, 2011).

$$Y(m) = a\left(1 - \frac{e^{\frac{c-m}{b}}}{2}\right)e^{-d\times m} \quad [5]$$

Using this function the 305 day estimated milk production (kg) were approximately 10000, 11,000 and 12,000 for the first 3 lactations, respectively.

4.3.4.2. *Live Body Weight*

Average monthly live weight for each state was calculated using Korver function (Korver et al., 1985) as described by (van Arendonk, 1984). Body weights were used to calculate the carcass value of the replaced cow and to estimate dry matter intake for each cow state.

4.3.4.3. *Dry Matter Intake*

Daily dry matter intake was calculated using Spartan 2 (Vandehaar et al., 1992) equation; which is a function of maintenance and milk production according to month in milk, m. This function used body weight and 4% fat corrected milk yield as inputs.

$$\boldsymbol{DMI_m} = (\mathbf{0.02} \times \boldsymbol{BW_m}) + (\mathbf{0.3} \times \mathbf{4\%}\boldsymbol{FCM_m}) \quad [6]$$

Where BW is the live body weight and 4%FCM is 4% fat corrected milk.

4.3.4.4. *Calf Value*

It was assumed that all 1 week old calves are sold and the value was assumed to be the weighted average of the value for male and female calves (Meadows et al., 2005).

4.3.4.5. *Involuntary Culling*

Cows at every state had the risk of being involuntary culled. The risk of involuntary culling was increased by lactation and MIM. Data from De Vries et al. (2010) was used to incorporate these transition probabilities.

4.3.4.6. *Reproduction*

Voluntary waiting period of 60 days (time when cows are eligible for insemination) and an 18% 21-day pregnancy rate were assumed. Cows were not bred anymore after 10 MIM (a.k.a., cut-off time). Pregnancy losses were included following (De Vries, 2006).

4.3.4.7. *Economic Parameters*

Replacement heifer cost was set at US$1,300/cow. Feed price for lactating and dry cows were set at US$0.22/kg and US$0.18/kg, respectively (Cabrera, 2012b). Other economic variables are summarized in Table 4.1.

4.3.5. Computer Implementation

The DP model as originally developed by Kalantari et al. (2010) was used to find the optimal replacement decisions. The Markov chain cow value model described by (Cabrera, 2012b) was re-coded as a standalone executable program with Visual Basic Net 2010 (Microsoft Corp., Redmond, WA).

Table 4.1. Economic variables (US$) used for both models: dynamic programming (DP) and Markov chain (MC)

Economic variables	Value
Replacement cost, $/cow	1,300
Carcass value, $/kg	0.38
Calf value, $/calf	100
Milk price, $/kg	0.35
Feed price for lactating cow, $/kg	0.22
Feed price for dry cows, $/kg	0.18
Interest rate, %/year	6

4.3.6. Model Comparison

The most important result of these two models was the ranking of all the animals in the herd according to their expected cow value or RPO. Therefore, cow value (calculated from Markov chain model) and RPO (from DP model) were used to rank animals and compare both models' results. The Spearman's rank correlation test was used to compare rankings from both models. The "spearman" package (Savicky, 2009) in R statistical software (R Development Core Team, 2011) was used to perform this statistical test.

4.3.7. Post Optimality Analysis

After finding the optimal decisions with a DP model, Markov chain models are used to find the herd demographics (herd structure) and economic parameters under optimal decisions. Three different scenarios were desgined to compare the effect of optimal decisions on the overall herd dynamics and herd net return. The first scenario used the Markov chain model as described in Cabrera (2012). The second scenario ran the Markov chain model under optimal decisions found by the DP model (De Vries, 2004; Kalantari et al., 2010). And the third scenario used a 2-step solution procedure of the Markov chain model. Negative values in the first solution were considered replacement decisions that were applied as optimal decisions for the second solution.

4.3.8. Sensitivity Analysis

Sensitivity analysis was later used to assess the effect of change of the main parameters on the accordance of the two models results. The most important factors affecting the culling decisions have been well studied and include milk production level and replacement cost (van Arendonk, 1985b; van Arendonk and Dijkhuizen, 1985; Kalantari et al., 2010). Therefore, the effect of 20% change in milk production level and 20% change in heifer purchase price were studied.

4.4. RESULTS and DISCUSSION

We first compare the similarities between the alternative methods used in this study. The cow value ranking accrued by solving both models had a strong linear relationship. Spearman's correlation (rho) between rankings of the 1,000 possible states was 89% (df=998, p-value < 0.0001). This correlation factor was affected by methodological differences between models, mostly regarding to the last lactation. In DP model, cows in their last lactation and late MIM were considered to be at their end of productive life and therefore replaced regardless of their pregnancy status. The keep value for these cows was calculated with a different equation than other cow states, i.e., Equation (5) in Kalantari and Cabrera (2012), which forces replacement of these cows. In fact, this forced replacement of DP formulation affects sequentially all lactations, but has the highest impact in the last lactation, because each value is dependent on the optimal decision of the next cow state in the previous stage. However, in Markov chain model the value of the cow was calculated the same way regardless of lactation, and there was no distinction between cow value calculations of different lactations. Under those circumstances, last lactation was excluded for further analyses. After this exclusion, Spearman's correlation increased to 95% (df=898, p-value < 0.0001). The weighted average cow value - estimated by Markov chain model and weighted by proportion of cows in different states - corresponded to this ranking was

$554 and in DP model the average RPO was $542. In both models the best ranked cow (highest positive value) was a fresh cow in third lactation -Markov chain with US$872 and DP with $917. Also, the least valuable cow was shared by both models as a cow in 9th lactation, last month in milk, and non-pregnant. DP model's RPO for this cow was - $44 and cow value in Markov chain was - $355. This negative RPO or cow value means that replacing a cow with a replacement heifer would be more profitable than keeping the cow one more month in the herd. The big difference in the magnitude of the values is due to the fact that DP follows optimal pathway and would not accumulate negative values. However, there is no optimal strategy in the Markov chain model.

A scatter diagram of the ranking of cow values in both models for 900 states over 9 lactations is shown in Figure 4.1 Rankings are closer at the beginning and at the end of the diagram. The diagram shows a bifurcation in the rankings and it is obvious that the rank for some cows does not follow the same pattern in both models. The upper groups of points in the diagram correspond to open cows in early lactation. However, these cows are far for being candidates for replacement.

The most important part of Figure 4.1, for practical decision-making and management, is the end tail of the graph (right top corner) that represents the lowest ranking cow states. These cow states with the lowest values are the most likely candidates for replacement decisions. The agreement (Spearman's correlation) between the two models was 98% on a state space represented by 10% of all cow states in the model. The percentage of negative values in the two models was not the same, i.e., 10% of all states in the DP model (corresponding to open cows >12 MIM in the first lactation and >10 MIM in other lactations), and 12% of all states in the Markov chain model (corresponding to open cows > 10 MIM in the first lactation, > 9 MIM in the second lactation, and >8 MIM in later lactations). Since voluntary replacement decisions will not exceed 4% of the herd in one month (Fetrow et al., 2006), this result indicates that final and practical replacement decisions are almost identical with both models.

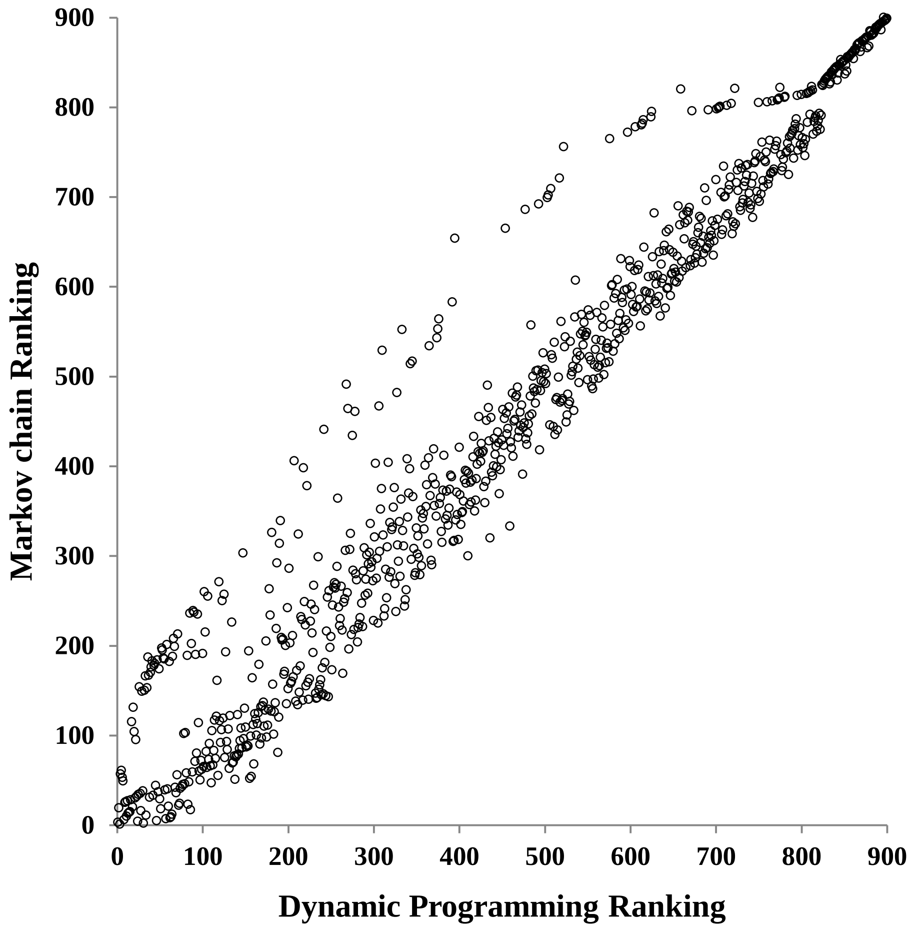

Figure 4.1. Relationship between ranking (higher to lower) from dynamic programming model (DP) retention pay off (RPO) and Markov chain model (MC) cow value over nine lactations (900 cow states)

Table 4.2 shows the breakdown of the overall correlation by pregnancy status, parity number and stage of lactation. Generally, all the correlation factors are greater than 90%, which indicates strong positive relationships between models' results. It should also be mentioned that different pregnancy status showed strong Spearman's correlation, which suggested that the models also had a high agreement based on pregnancy status.

Table 4.2. Spearman's correlation (rho) between dynamic programming model (DP) retention pay off (RPO) and Markov chain model (MC) cow value broken down by pregnancy status, parity and stage of lactation with number of pair observations from models (n) at each state.

States	rho	States	rho
Open (n=171)	0.995	3rd Parity (n=100)	0.968
1st MIP[1] (n=81)	0.970	4th Parity (n=100)	0.964
2nd MIP (n=81)	0.976	5th Parity (n=100)	0.957
3rd MIP (n=81)	0.982	6th Parity (n=100)	0.954
4th MIP (n=81)	0.989	7th Parity (n=100)	0.955
5th MIP (n=81)	0.994	8th Parity (n=100)	0.957
6th MIP (n=81)	0.992	9th Parity (n=100)	0.951
7th MIP (n=81)	0.966	Early lactation (MIM[2]=1,2) (n=18)	0.742
8th MIP (n=81)	0.881	Mid lactation (MIM=3-8) (n=243)	0.838
9th MIP (n=81)	0.916	Late lactation (MIM=9-14) (n=459)	0.978
1st Parity (n=100)	0.964	Very late lactation (MIM=15-19) (n=180)	0.995
2nd Parity (n=100)	0.973		

[1]MIP= month in pregnancy, [2]MIM= Month in milk, n=number of observation with the specified state

4.4.1. Post Optimality Analysis

Post optimality analyses are summarized in Table 4.3. The first scenario that used a Markov chain without any optimal decisions reported a net retun of $1,584/cow per year. The net return under optimal decisions from DP was $6/cow per year higher than the Markov chain without optimal decisions. As expected, this difference was mostly originated from reduced culling costs. Therefore, changing replacement policies according to DP results would equate in extra US$6/cow per year.

The net return resulting from Markov chain with suboptimal decisions (2-step solution scenario) was equal to the one using the DP's optimal decisions, although there were slight differences in specific economic components. Main differences between these two scenarios occurred in milk income and culling costs. Culling cost in the Markov chain model was mainly affected by applying the cut-off at 10 MIM and also having 2% more non reprodutive culling than the DP optimal decisions. Although the cut-off MIM applied equally in both models, this cut-off in the Markov chain model indicated replacement for these cows (reproductive culling). However, in DP model, cut-off MIM only meant a different calcuation of the keep value, which did not include reproductive service costs, (Kalantari and Cabrera, 2012). Another source of net return difference between the 2-step Markov chain and the DP model was higher milk sales in the Markov chain model. This difference was also related to the cut-off MIM. Cows were culled at 10 MIM in Markov chain model, which resulted in a slightly different herd structure (more early lactation cows) that yielded increased total milk revenue Table 4.3.

Table 4.3. Economic parameters and herd structure resulting of Markov chain model simulations under different scenarios

Scenario	Economic Parameters (US$/cow per yr)						Herd structure						
	Net return	Milk sales	Feed cost	Calf sales	Cull cost	Rep cost	Lact 1 %)	Lact 2 (%)	Lact 3 (%)	Lact ≥ 4(%)	DIM (d)	Preg. (%)	Lact (%)
MC[1]	1,584	3,266	-1,402	63	-274	-69	34.38	25.4	16.69	23.2	138	60.8	81.22
MC+DP[2]	1,590	3,263	-1,401	63	-265	-69	34.84	25.26	16.59	23.04	141	60.53	81.48
MC+S[3]	1,590	3,279	-1,400	63	-280	-71	36.28	26.27	16.46	20.99	135	60.6	81.23

[1] Markov chain simulation without optimal decisions

[2] Markov chain simulation with optimal decision obtained from DP

[3] Markov chain simulation with suboptimal decisions obtained from Markov chain

Herd structure and dynamics at steady state of the 3 scenarios studied are also summarized in Table 4.3. The Markov chain and DP model's overall herd structures are not substantially different. However, results from the Markov chain under suboptimal decisions (2-step solution) showed discrepencies with results of both the original Markov chain and the DP model. The most important difference was a 1.44% change in the proportion of cows in the first parity in favor of the Markov chain with suboptimal decisisions. This difference could be attributed to higher culling rates (mainly reproductive culling) in this scenario.

4.4.2. Sensitivity Analysis

Twenty percent changes in the milk production and heifer price did not affect the overall correlation factor of two models, remaining greater than 90% in every scenario. The effect of these changes on cow value is illustrated in Figure 4.2. Because the optimal pathway is followed in DP model through iterations, not much negative values are accumulated and the minimum observed was -US$44. The dispersion of cow values in the Markov chain model was higher than in the DP (Figure 4.2).

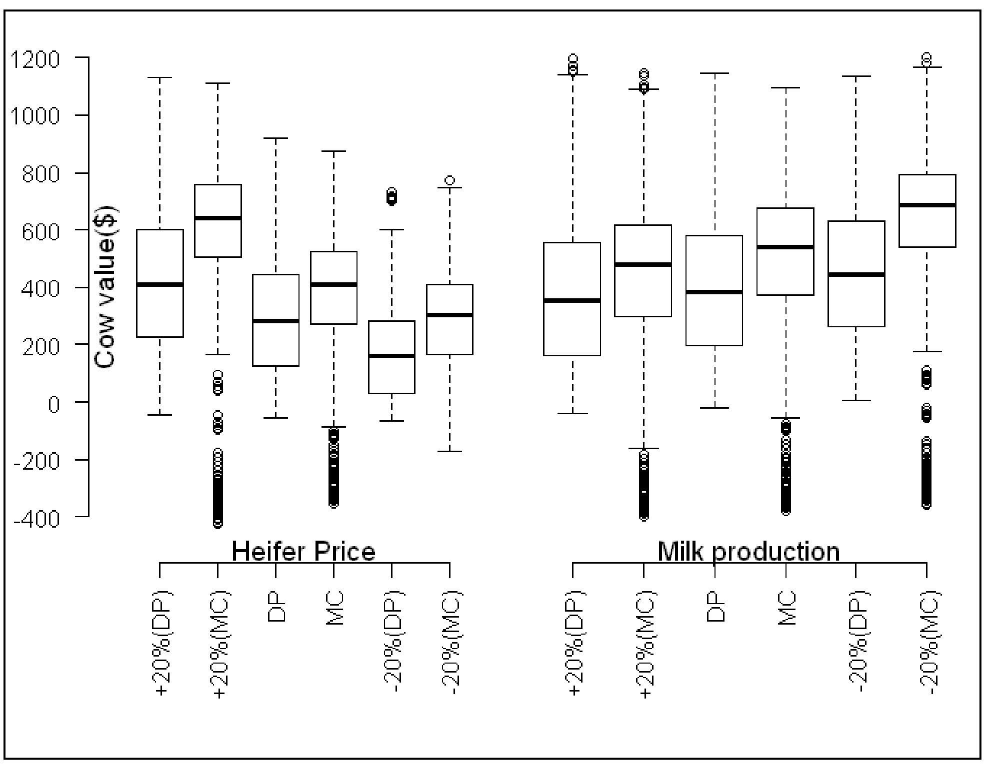

Figure 4.2. The cow value (US$) from dynamic programming model (DP) and Markov chain model (MC) for a 20% change (from baseline scenario) in heifer price and milk production

4.4.3. Computer Implementation of the Models

Both models have some advantages and disadvantages regarding computer programming. Despite the fact that today's computers are every time faster and more powerful, complexity of DP models in problem formulization and conceptualization (Burt, 1982) are still limiting, mostly when the main aim is to develop practical decision-making and management applications. Markov chain cow value models are easier to implement and simpler for computer programming.

Culling decisions in farms are usually for a few candidate cows for replacement, and therefore, models should evaluate the cow value just for those cows. Inability to calculate directed cow-specific RPO is another disadvantage of the DP model when thinking on computational easiness and practical decision-making. The DP model needs to calculate the keep and replace values for all the possible states in the problem within a solution. The Markov chain model can easily overcome this problem by targeting most likely replacement animals, saving great amount of computational time.

On the other hand, calculation of the value for all cow states is a major advantage of the DP model over the Markov chain cow value model. In order to evaluate and compare the results of both models in this study, assessment of all the possible states was necessary with the Markov chain model, which, at the end, took more computational time than the DP model. Nevertheless, this longer time could have been substantially reduced by using parallel programming techniques, which take advantage of multi-core processors (Ostrovsky, 2010). Parallel programming is not an option for DP because of its stepwise intrinsic nature. That is, each iteration evaluation depends on the results from the previous iteration. The new Markov chain model is a perfect candidate for running in parallel because each state evaluation is completely independent from the others.

4.5. CONCLUSIONS

We found a strong correlation (95%) on replacement decisions resulting from using two completely different modeling approaches: The classical and state-of-the-art dynamic programming framework and a newly developed technique using simple simulation of Markov chains. Post optimality analyses demonstrated that overall long-term herd structure and herd net returns resulting from models' replacement policies were very similar. These results strongly support that the newly developed Markov chain is a good alternative for practical dairy decision-making and for the development of decision support systems.

4.6. ACKNOWLEDGMENTS

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Chapter 5

Stochastic Economic Evaluation of Dairy Farm Reproductive Performance

Stochastic economic evaluation of dairy farm reproductive performance

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5.1. ABSTRACT

The objective of this study was to assess the economic value of reproductive performance in dairy farms under uncertain and variable conditions. Consequently, the study developed methods to introduce stochasticity into transition probabilities of a Markov chain model. A robust Markov chain model with 21-day stage length and 3 state variables -parity, days in milk, and days in pregnancy- was developed. Uncertainty was added to all transition probabilities, milk production level, and reproductive costs. The model was run for 10,000 replications after introducing each random variable. The expected net return ($/cow per year ± standard deviation) was $3,192±75.0 for the baseline scenario that had 15% 21-day pregnancy rate (21-d PR). After verifying the model's behavior, it was run for 2000 replications to study the effect of changing 21-d PR from 10 to 30% with one-unit-percentage interval. The economic gain of changing 21-d PR from 10% to 30% resulted in a $75 per cow per year, and this overall increase in the net return was observed mainly due to the lower reproductive and culling cost and higher calf value. The gain was even greater when milk price and milk cut-off threshold decreased.

Key words: Markov chain, randomness, reproductive performance, economics

5.2. INTRODUCTION

Reproductive performance affects dairy herd profitability either directly through modifying the number of calves produced or sold (Giordano et al., 2012), or indirectly by changing the distribution of cows throughout lactations (Kalantari and Cabrera, 2012). According to dairy herd improvement (DHI) benchmarks (AgSource Cooperative Services, 2013) the average pregnancy rate for the Holstein dairy farms in Wisconsin with 100-250 cows is only 14.6%, and there is a 9 percentage points difference between the 80^{th} (19%) and 20^{th} (10%) percentiles. Many studies (Olynk and Wolf, 2009; Inchaisri et al., 2010; Giordano et al., 2012; Galvão et al., 2013) have attempted to quantify the economic impact of reproductive programs on defined dairy farms. However, reproductive performance might affect farms' profitability differently. Farms with similar reproductive performance could expect different profits from similar reproductive programs. Many important stochastic factors should be considered simultaneously to analyze the effect of reproductive performance among different farms (Olynk and Wolf, 2009). Simulation models can help to better examine the system behavior under uncertain conditions (Allore et al., 1998b). Markov chain simulation model has been used to simulate dairy farm systems and study reproductive decisions in dairy cattle (Jalvingh et al., 1993a; St-Pierre and Jones, 2001; De Vries, 2004; Cabrera et al., 2006; Giordano et al., 2012), to evaluate the expected value for the herd structure and economics at steady state. This model also can be used to estimate biological variation among cows in a herd at steady state. This variation is due to transition probabilities (probability of pregnancy, culling, death, and abortion), which introduces variation among cows in a herd based on their current state and the chance of moving to another state based on the transition probabilities. However, the Markov chain model does not include uncertainty in the input parameters due to imperfect knowledge; therefore, the model produces expected values for all the outputs given predefined input parameters (Jalvingh, 1992). Hence, an adjusted Markov chain model could be used to study the uncertain economic value of alternative reproductive programs. This method follows the concept of robust optimization used in the operations research literature (Iyengar, 2005). For example, robust dynamic programming considers sets of possible transition probabilities to account for uncertainty in the transition probabilities to capture its effect on the optimal policies being made by the model (Iyengar, 2005). The robust Markov chain model developed and presented in this study was envisioned to include such uncertainties and randomness that could be expected within and between targeted dairy farms.

Including stochasticity into input parameters (transition probabilities) of a Markov chain model produces uncertainty around the outcomes, both herd economics and dynamics. Presenting the results with uncertainties in conjunction with knowledge of farmers' risk preferences can be useful to make better-informed decisions (Olynk and Wolf, 2009). Consequently, this might be helpful to direct their management practices to higher profitability given their current reproductive performance. More specifically, distribution in the outcomes could quantify the probability of reaching a target net return, thus, giving decision-makers a useful cue in directing their management practices to attain higher profitability given their current reproductive performance. These opportunities are not available with a standard Markov chain.

Thus, the main aim of this study was to evaluate the economic impact of changes in reproductive performance under uncertain conditions of milk productivity, involuntary culling, abortion, pregnancy rate, and reproductive costs using robust Markov chain model.

5.3. MATERIALS AND METHODS

5.3.1. Dairy Farm Specification

A Holstein dairy herd with an average of 150 cows located in Wisconsin or in the Midwest US was defined and studied. The milk production level was set to be approximately 10,000 kg/year per cow and the average annual herd turnover rate to 35% based on dairy herd improvement benchmark data (AgSource Cooperative Services, 2013). The cows in the herd were fed a single total mixed ration throughout their lactation. Other average herd and economic variables used in the study are summarized in Table 5.1.

Table 5.1. Average[1] input variables of the model. Economic values are in US dollars

Variables name	Average value	Source
Input herd variables		
Herd turnover (%/yr)	35	De Vries et al. (2010)
Milk production level (kg/yr)	10,000	DHI benchmark[2] (2013)
Dry period (d)	60	DHI benchmark[2] (2013)
Last day to breed a cow (d)	294	Giordano et al. (2012)
Milk threshold (kg/cow per day)	23	Giordano et al. (2012)
Pregnancy loss (%/lactation)	8.2	De Vries (2006)
21-day Pregnancy rate (%/yr)	14.6	DHI benchmark[2] (2013)
Mortality[3] (%/yr)	20	Pinedo et al. (2010)
Economic variables		
Replacement cost, ($/cow)	1,300	
Carcass value, ($/kg)	0.38	
Calf value, ($/calf)	100	Cabrera (2012)
Milk price, ($/kg)	0.43	
Feed price for lactating cow ($/kg DM)	0.22	
Feed price for dry cows ($/kg DM)	0.18	

[1] Average herd variables were used as a baseline in the model and uncertainty was built around these averages.

[2] Based on dairy herd improvement benchmark provided by AgSource Cooperative Services (2013) average values for herds with average 150 cows.

[3] As a percentage of total culling.

5.3.2. Markov Chain Model

A robust Markov chain model (De Vries, 2004; Cabrera et al., 2006; Giordano et al., 2012) was developed and used to simulate dairy herd structure and economics at steady state. The model simulated the herd using 3 state variables (Giordano et al., 2012; Kalantari and Cabrera, 2012): lactation number (l = 1 to 9), days in milk (**DIM**; d = 1 to 735 with 21-d steps), and days in pregnancy (p = 0 to 282; 0 for open cows and 1 to 282 with 21-day steps). The model used the average reproductive cycle of dairy cows of 21 d as the stage length to better simulate reproductive cycles. Therefore, all the events such as: involuntary culling, abortion, and pregnancy were adjusted to 21-d cycles.

Transition probabilities were used to change the state of the cow from one stage to the next (every 21 d); which meant cows followed probabilistic events of aging, replacement, dying, getting pregnant, aborting, calving, and starting a new lactation (Giordano et al., 2012). Therefore, the herd structure- the proportion of cows available at each state- at steady state was driven by these transition probabilities. The simulation started by placing a group of cows at the first lactation, first DIM and non-pregnant (a recently calved heifer) and continued by moving it forward through all the defined states until the model reached steady state, which occurred when the proportion of cows did not change from one stage to the next. The proportion (P) of cows in each state and in the next stage was calculated as following (using Giordano et al. (2012) notation):

For open cows (p = 0) eligible for insemination (42 ≤ d ≤ 294).

And becoming pregnant:

$$P_{r,l,d+21,1} = P_{r,l,d,0} \times \left(1 - PleaveO_{r,l,d}\right) \times Ppreg_{r,l,d}, \quad [1]$$

Not becoming pregnant:

$$P_{r,l,d+21,0} = P_{r,l,d,0} \times \left(1 - PleaveO_{r,l,d}\right) \times (1 - Ppreg_{r,l,d}), \quad [2]$$

For open cows (p = 0) and not breeding eligible (294 < d ≤ 735):

$$P_{r,l,d+21,0} = P_{r,l,d,0} \times \left(1 - PleaveO_{r,l,d}\right), \quad [3]$$

For pregnant cows and not aborting:

$$P_{r,l,d+21,p+1} = P_{r,l,d,p} \times \left(1 - PleaveP_{r,l,p}\right) \times (1 - Pabort_{r,p}), \quad [4]$$

Aborting cows:

$$P_{r,l,d+21,0} = P_{r,l,d,p} \times \left(1 - PleaveP_{r,l,p}\right) \times Pabort_{r,p}, \quad [5]$$

For cows calving and moving to the next parity:

$$P_{r,l+1,1,0} = \sum_{d=VWP+gest}^{repcull+gest} P_{r,l+1,1,0} + \left(P_{r,l,d,282} \times \left(1 - PleaveP_{r,l,282}\right)\right), \quad [6]$$

Where l = lactation number, d = DIM with 21 d steps, p = pregnancy and r = replication number; $PleaveO_{r,l,d}$ and $PleaveP_{r,l,p}$ are transition probabilities of leaving the herd for non-pregnant and pregnant cows (due to non-reproductive and reproductive replacement and mortality); $Ppreg_{r,l,d}$ is the probability of becoming pregnant; $Pabort_{r,p}$ is the probability of pregnancy loss; *repcull* is the cut-off DIM for breeding in days; *VWP* is the voluntary waiting period in days; and *gest* is the gestation length in days. Following the main Markov chain assumption of a constant herd size (De Vries, 2004), all cows leaving the herd at any stage were replaced with recently calved heifers $P_{r,1,1,0}$ in the next stage. A critical difference between the current model and the model described in Giordano et al. (2012) is the replication subscript in each component of the model. This gives the current model capability of running different replications of farms (based on different input parameters such as milk production level, involuntary culling and abortion) independently and concurrently. This replication subscript allows the model to use different input parameters for each replication, which is unique in this study. The random module is described in the following section.

After finding the herd structure at the steady state, the net return ($/cow per day) for each replication (r) was calculated as follows:

$$NR_r = \sum_{l=1}^{9}\sum_{d=1}^{35}\sum_{p=0}^{13} P_{r,l,d,p} \times [IOFC + CV - CC - RC]_{r,l,d,p}\ , \qquad [7]$$

Where NR_r is the net return for each replication r; IOFC is milk income over feed costs; CV is income from calves, CC is the culling cost- which is the difference between the salvage value of the culled cow and the replacement heifer price- and RC is the average reproductive cost (Table 5.1). Then, Eq. 7 calculates the overall net return of each replication in the model by weighting the economic parameters by the proportion of cows at each state. Furthermore, one can find variation among the cows in every replication (herd), which is due to the observed variation among cows in a herd and can be calculated as follows:

$$Var_r = \sum_{l=1}^{9}\sum_{d=1}^{35}\sum_{p=0}^{13} P_{r,l,d,p} \times \left[(IOFC + CV - CC - RC)_{r,l,d,p}\right]^2 - (NR_r)^2\ , \qquad [8]$$

This is the variation due to the biological variations among cows in a herd; therefore it is mainly affected by factors that change the herd structure or proportion of cows at different states within a herd ($P_{r,l,d,p}$). The formula calculates the average net return squared deviation of each cow state in a replication given the probability weight of being in each state. This variation was not the main purpose of this paper, but was included in the analyses for completeness.

5.3.3. Random Module

Uncertainty was added to milk productivity and transition probabilities- including involuntary culling, reproductive performance, and abortion- in the Markov chain model using a stepwise approach. A stepwise approach was useful to verify the model, and also to study the result of introducing a single random parameter on the herd statistics and economics. Random number generation process followed a synchronized common random numbers method, which resulted in variance reduction of the point estimators between replications of different samples (Banks et al., 2009). In this method, the same stream of random numbers (common random numbers) are used for the same parameters (synchronized) from one run to another, which helps to reduce the variation due to the different random numbers between replications (Banks et al., 2009). Results are more precise due to variance reduction. Also, computational time is reduced because of fewer replications. Each replicate of the model was run to steady state with the same stream of random numbers, which intended to simulate different herd conditions (different farms). Finally, the produced randomness was added to the average values by introducing white noise to some of the transition probabilities, which followed a systematic approach as described below.

A polynomial regression model was fitted to average data from historical data. After that, the predicted value of the fitted model was calculated and the residual of this prediction was used to build a random spectrum around each data point. This random number was modeled using a normal distribution with mean of zero and the standard deviation equal to that of the residual from the fitted model (N~(0, SD residual)). By using this method, the distribution of values over time followed the observed pattern in data and with an extra random error around the average observed values from historical data. As an example, a complete procedure for this method of modeling for involuntary culling is described later. For reproductive performance, a triangular distribution was used to generate most plausible random variables within ranges- based on minimum, maximum and most likely values of pregnancy rates. A normal distribution was used for introducing variation in milk production levels within and between herds.

5.4. Stochastic Parameters

5.4.1. Milk Production

The MilkBot function (Ehrlich, 2011) was used to fit milk production curves. The MilkBot predicts milk yields (Y) as a function of time after parturition. Four parameters: a (scale), b (ramp), c (offset), and d (decay), control the shape of the lactation curves (Ehrlich, 2011).

$$Y(m) = a\left(1 - \frac{e^{\left(\frac{c-m}{b}\right)}}{2}\right)e^{(-d\times m)} \ , \qquad [9]$$

Eq. 9 was used to describe milk production curves for the first, second, and ≥ 3 lactations from DHI test days with average milk production level of 10,000 kg/year per cow. Then, milk levels were built around this average by using a modified version of van Arendonk (1985) approach. Van Arendonk's approach assumed that total milk production followed a normal distribution with standard deviation of 10%. In the current study, the standard deviation was chosen in such a way that the variation covered middle 3 quintiles based on the DHI records (AgSource Cooperative Services, 2013). Therefore, a 2% standard deviation was used to represent the variation observed between farms with the average of 10,000 kg/year per cow (AgSource Cooperative Services, 2013). A 2% standard deviation resulted in a variation of about 1,300 kg/year per cow in herd's overall milk yield. Fifteen milk classes were built around the average milk production. These 15 milk classes represented different farms' milk productions, and the probability of being at each milk class followed a distribution of N~(100, 2). These probabilities were used to generate random numbers for 15 milk classes. Then, each class represented an average milk production level from a farm. To generate different milk production levels the average milk production were set at the 100% (average) and all other milk productions levels were calculated relative to this average. For example, milk class 1 produced 93% of the average and the probability of being in this class was 0.06%. Milk production curves are illustrated in Figure 5.1 for open cows at their first and second lactations for 7% below and above average as described above. Other milk production levels and probabilities were calculated similarly. Moreover, a random dependent milk production error was added based on Allore et al. (1998) and De Vries (2001) to capture the possible fluctuation within each milk class (variation among cows).

$$M_t = 0.9896 \times M_{t-1} + e_t, \qquad [10]$$

Where M_t is a random dependent milk production error and e_t is the random independent residual on t days in lactation and followed $e_t \sim N(0, 0.45^2)$. Milk production was also adjusted to decrease by a fixed factor of 5, 10 and 15% by month of pregnancy 5, 6, and 7, respectively, based on (De Vries, 2004). Daily milk production then was calculated by summing up average milk production from Eq. 9, random error from Eq. 10, and possible pregnancy milk depression based on fixed factors, always with the consideration of the milk class.

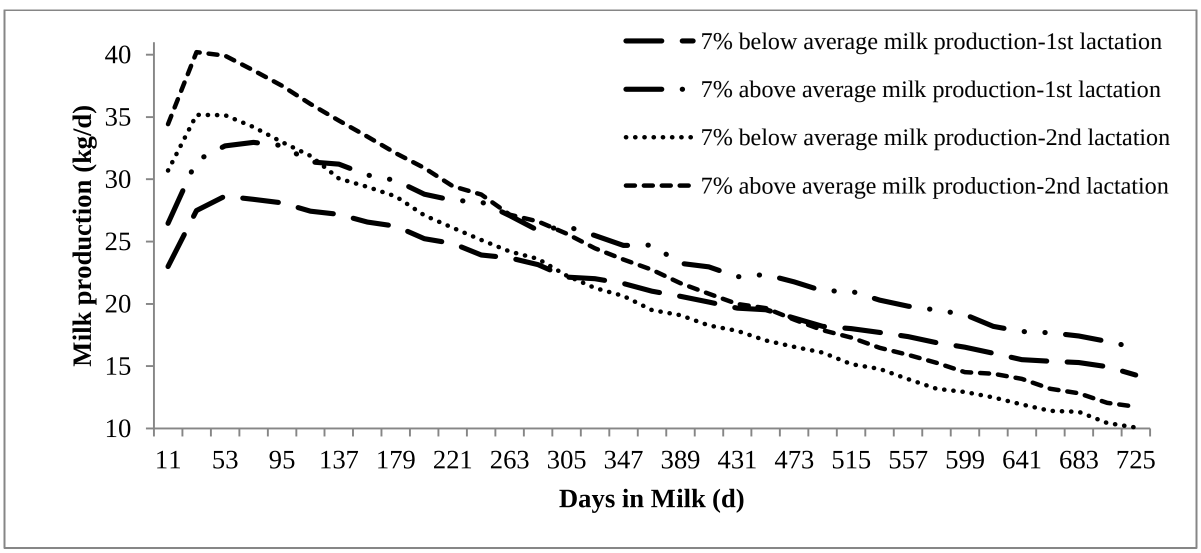

Figure 5.1. Milk production curves for open cows in their first and second lactations for 7% below and above average milk production curve. 7% below and above average represents the lowest and highest milk production levels, respectively, relative to the average milk production.

5.4.2. Involuntary Culling

Cows in every state had a risk of involuntary culling. This risk increased by lactation and DIM. The pattern of involuntary culling throughout the lactation was based on data from (De Vries et al., 2010a). A polynomial regression model was used to fit a model to the historical data. The residual standard deviation of the predicted data (from the regression model) and the actual data was used to generate white noise bounds around involuntary culling curves across days in milk.

The binomial family of the GLM method in R statistical software (R Development Core Team, 2011) was used to fit a 5^{th} order polynomial regression on the observed probabilities of involuntary culling. The binomial family in this method corresponds to using logistic regression. The predicted values from this model were in logit scale and required final rescaling. The regression model was extracted and used for creating the bounds by incorporating the normal distribution of mean zero and residual's standard deviation. The following equation, for example, is the fifth-order polynomial regression model used to predict the risk of involuntary culling for second lactation and non-pregnant cows.

$$InvCull_d = -3.5 - 0.28d + 0.075d^2 - 0.004d^3 + 0.00001d^4 - 0.000001d^5, \qquad [11]$$

Where InvCull is the predicted value for the risk of involuntary culling on the logit scale and it depends on the DIM (d). Finally, the residuals were used to build random bounds around the predicted value. After finding the predicted value by adding the random part to the resulted value from Eq. 11, the rescaling was done by back-transforming the logit using the following equation:

$$invlogit = \exp(InvCull)\,/(1 + \exp(InvCull), \qquad [12]$$

A sample of 1,000 random numbers of percentages of risk of involuntary culling that was generated using this method is presented in Figure 5.2. The bounds were truncated to eliminate the possibility of negative involuntary culling due to generated random numbers. The same procedure was used to generate random numbers for the first 3 lactations. For lactations ≥ 4, the same baseline numbers were used, but the random numbers were generated for each lactation separately. The risk of involuntary culling of pregnant cows was set at 25% of non-pregnant cows (De Vries et al., 2010a). Mortality baseline rates were set as 20% of all the culling according to (Pinedo et al., 2010). The same procedure was applied to generate random numbers and create bounds around each data point for involuntary culling of pregnant cows and mortality rates.

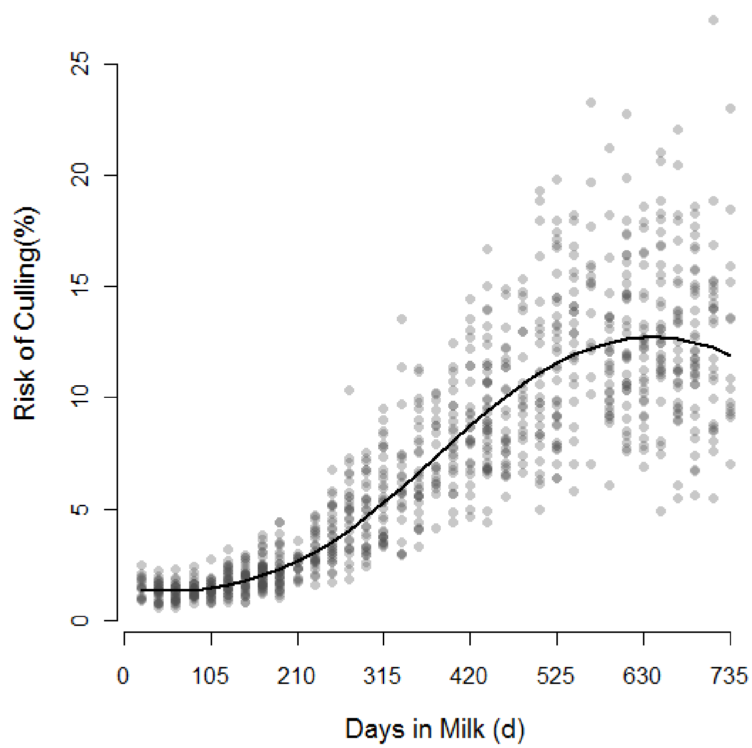

Figure 5.2. Example of 1,000 random numbers used to introduce uncertainty to the risk of culling of second lactation cows throughout 735 day in milk. Black line represents average risk of involuntary culling for the second lactation and dots are sample of random numbers around this average.

5.4.3. Reproduction

Voluntary waiting period was assumed to be 42 days to follow the 21-day stage length of the model. Cows were eligible for insemination from 43 to 294 DIM. After this cut-off point cows were labeled as do-not-breed, and they were culled when their average milk production dropped below 23 kg (culling because of reproductive failure). The 21-day pregnancy rate (**21-d PR**) was used as a metric of reproductive efficiency. The 21-d PR measures the rate at which eligible cows become pregnant every 21 days (Ferguson and Galligan, 1999). In order to capture the possible variation among breedings between and within lactations, a triangular distribution was used to create random variables for 21-d PR. Parameters of this triangular distribution were chosen in such a way that the most likely value represented the average of 21-d PR (input variable) and the minimum and maximum were 5 percentage point above and below this average. This 5% range was chosen to approximate the difference existent between 20^{th} and 80^{th} percentile, 10% and 19% 21-d PR, respectively, around the 21-d PR average of 14.6%, according to Wisconsin dairy herd improvement benchmarks (AgSource Cooperative Services, 2013). The assumption was that a similar range of 21-d PR could be expected within a farm from one breeding period to the next. This introduced an independent variation among pregnancy rates both between and within lactations. Historical data for pregnancy losses was based on De Vries (2006) and the process for introducing randomness around pregnancy losses was the same as the one described for involuntary culling.

Authors did not find any published work that compares the average reproductive costs of different 21-d PR. It is reasonable to assume that greater reproductive investments are needed to achieve higher reproductive performance. Hence, it was assumed that herds with a better reproductive performance have invested more on management and facilities, which resulted in better detection of estrous or better overall conception rates. Thus, based on the estimated average 21-d PR, different reproductive costs ($/21-d) were assigned to herds. For this purpose two triangular distributions were parameterized based on the 21-d PR of herds and baseline reproductive costs recently reported (Cabrera, 2014). Reproductive cost for farms with < 20% 21-d PR used a triangular distribution with minimum, mode and maximum of 15, 20, and 25 US dollars per 21 day, respectively. The parameters for 21-d PR above 20% were set to 20, 25, and 30 US dollars per 21 day for minimum, mode, and maximum, respectively.

5.5. Deterministic Parameters

5.5.1. Live Body Weight

Average monthly live weight for each state was calculated using the Korver function (Korver et al., 1985) as described by Van Arendonk (1985). Body weights were used to calculate the carcass value of the replaced cow and to estimate dry matter intake for each cow state.

5.5.2. Dry Matter Intake

Daily dry matter intake was calculated using Spartan 2 (Vandehaar et al., 1992) equation, which is a function of maintenance and milk production. This function used body weight and 4% fat corrected milk yield as inputs.

$$DMI_m = (0.02 \times BW_m) + (0.3 \times 4\%FCM_m), \quad [3]$$

Where DMI is dry matter intake, BW is the live body weight, and 4% FCM is 4% fat corrected milk and m is the 21-d stage length of DIM. Cows were fed a total mixed ration based on their dry matter intake and different fixed costs ($/kg; Table 5.1) were used for lactating and dry cows.

5.5.3. Calf Value

It was assumed that all calves were sold one week after they were born and the value was assumed to be the weighted average of the value for male and female calves (Meadows et al., 2005).

5.5.4. Economic Parameters

The study was performed with given prices and costs summarized in Table 5.1. With the exception of reproductive costs, other economic parameters were not added as stochastic elements into the model.

5.5.5. Sensitivity Analyses

After obtaining the results using the default values from Table 5.1, sensitivity analyses were conducted to assess the effect of changes in economic parameters (replacement cost and milk price) on the economic value of a given level of reproductive performance. These economic parameters were chosen based on their importance on the economic value of reproductive performance based on previous studies (Galvão et al., 2013; Cabrera, 2014). Furthermore, the cut-off threshold of milk production for culling was reduced by half to explore the effect of lower threshold on the economic difference among reproductive performance levels. An interaction of replacement cost and lower milk cut-off threshold was also explored.

5.6. Computer Implementation

The robust Markov chain model was coded as a standalone executable program with Visual Basic Net 2010 (Microsoft Corp. Redmond WA). It was developed using parallel programming approach (Ostrovsky, 2010) to take advantage of the power of modern multi-core computer systems. Therefore, different runs on the random number streams were running independently at the same time (r subscript in the Markov chain model section represents each of this independent runs).

5.7. RESULTS AND DISCUSSION

5.7.1. Base Run Results

Herd's economics and dynamics from 10,000 replications before and after introducing random parameters for herds with 15% 21d-PR are summarized in Table 5.2. Without including any stochasticity in the input parameters, outcomes are the expected value estimated by initializing transition probabilities, milk production and reproductive costs by averages. The expected value of biological variation (Eq. 8) due to variability of the cow states within a herd for this scenario was $1,401/cow per year. This is the variation in the net return observed among all the cows in the herd and is mainly affected by input parameters that change the herd structure (e.g., involuntary culling, pregnancy rate). For example, adding involuntary culling as a random variable into the model increased this variation by $5/cow per year.

The second type of variation was the observed variation between different replications (herds) due to uncertainty in the input parameters, which is presented as the standard deviation of each parameter in Table 5.2. After step by step inclusion of each parameter as a random variable, the corresponding parameter in the herd's economics was affected (Table 5.2). For example, after introducing pregnancy rate as a random variable, its standard deviation changed and also the standard deviation of other factors that are affected by pregnancy rate. This occurred, with different magnitude, similarly after introducing other random variables. The increment in standard deviation was dramatic after introducing milk production as a random variable. Standard deviation of net return after including milk production jumped to $75. The higher impact of milk production compared to the rest of the variables is a result of greater standard deviation of milk production in the input parameters. Overall, it is clear that at 15% 21-d PR level, tremendous economic variation is expected around the average value of $3,192/cow per year (Table 5.2). It should be noted due to the usage of synchronized common random number, this observed variation is not because of the random variations between different random numbers among replications, but solely due to the original variations among input parameters (which would represent different farms).

The resulting herds' dynamics are comparable to recent studies using a Markov chain model (Kalantari et al., 2010; Giordano et al., 2012; Kalantari and Cabrera, 2012). Also, herd dynamics and reproductive characteristics (such as average DIM, open days) are in line with previous study (Giordano et al., 2012). However, there are some differences among the evaluation of the economic parameters from the current model and the one used in Giordano et al. (2012). These can be attributed to the stage length difference (21-d in current study vs. 1 d in Giordano et al. (2012)) and the way each parameter was modeled in these two models. For example, Giordano et al. (2012) aimed for very detailed construction of the reproductive programs including all the specifics related to reproductive cost evaluation (such as cost of labor for estrus detection and injection, hormones for synchronization and pregnancy diagnosis) and daily reproductive dynamics of the herd. In the current study, reproductive cost was included as a random parameter into the model and the assumption was that the herd with good reproductive performance has invested more. Reproductive costs ($/cow per year) in Giordano et al. (2012) were approximated $40, much less than $114 in the current study.

Table 5.2. Stepwise introduction of stochasticity into variables of the model and its effect on herd's economics and dynamics. Expected value (EV) ± standard deviations based on 10,000 replications run with stochastic Markov chain model.

	Runs						
Parameters	EV[1]	Inv[2]	Ab[3]	PR[4]	RC[5]	MP[6]	Allin[7]
<u>Economics</u> ($/cow per year)							
Net return	3192	±2.74	±0.01	±2.34	±11.58	±73.85	±75.0
Milk sales	4232	±0.98	±0.06	±2.6	0	±82.23	±82.33
Feed costs	-809	±0.17	±0.01	±0.71	0	±7.09	±7.13
Calf sales	63	±0.04	±0.02	±0.66	0	±0.12	±0.63
Culling cost	-179	±2.69	±0.03	1.71	0	±1.59	±3.55
Reproductive cost	-114	±0.08	±0.01	±1.31	±11.58	±0.21	±11.67
<u>Herd Dynamics (%)</u>							
Parity 1	37.2	±0.29	±0.01	±0.55	0	±0.05	±0.63
Parity 2	24.4	±0.16	0	±0.29	0	±0.04	±0.33
Parity 3	16.0	±0.13	0	±0.22	0	±0.01	±0.23
Parity >=4	22.4	±0.51	±0.02	±0.53	0	±0.02	±0.51
DIM (day)	180	±0.33	0	±1.2	0	±0.53	±1.4
Total Leaving[8]	40.0	±0.61	±0.01	±0.47	0	±0.64	±1.04
21-d Pregnancy rate	15	0	0	±0.7	0	0	±0.7

[1] Expected value (EV) without including any randomness based on basic Markov chain model

[2] Adding involuntary culling (Inv) as a random variable

[3] Adding abortion (Ab) as a random variable

[4] Adding 21-d pregnancy rate (PR) as a random variable

[5] Adding reproductive cost (RC) as a random variable

[6] Adding milk production (MP) as a random variable

[7] Adding all the previous variables into the model

[8] Total percentage of cows leaving the herd due to involuntary culling, reproductive culling, and mortality.

Besides these differences, the robust Markov chain presented here produced a margin of uncertainty for statistics in the results, which were not available in the previous Markov chain simulation studies. This margin can play an important role in final decisions made by farmers. For example, if the potential net return is high, but with a high margin of uncertainty, risk averse farmers might choose not to try to change their current reproductive management strategies, whereas risk taker farmers might still consider it.

Another approach that has been used to simulate dairy farms and evaluate economic impact of different reproductive performance is the stochastic dynamic model based on the Monte Carlo approach (Olynk and Wolf, 2009; Inchaisri et al., 2010; Galvão et al., 2013). This method was recently used by Galvão et al. (2013) to compare different reproductive programs. As expected, there are some differences among the economic evaluations in the current model and Galvão et al. (2013). These differences can be attributed to the different input variables (both economics and transition probabilities) in the two studies. For instance, the replacement costs (>$400/cow per year) and feeding costs (>$1700/cow per year) in Galvão et al. (2013) study were much higher than in this study.

The model developed in this study was also used to produce replications for different 21-d PR with one unit-percentage interval from 10 to 30%. Two thousand replications for each 21-d PR were simulated and the net returns ($/cow per year) were summarized as a box plot in Figure 5.3. Net returns at each 21-d PR point were approximately normally distributed indicated by medians and means values being the same (Figure 5.3). The net return increased as 21-d PR increased up to 21-d PR of 21%, after this point the assumption of more investments for achieving higher pregnancy rates caused a drop in the average net return. The higher reproductive cost counteracted the better reproductive performance until 25% 21-d PR. After this point the average net return showed a linear increase until 29% 21-dPR. A decreasing marginal return was not observed in this study because of using different reproductive costs for different reproductive performance and also because of having uncertainty in reproductive costs. However, it was observed that the gain from improving the 21-d PR is more in lower 21-d PR. This economic gain ($/cow per year) per each one percentage point from 10% to 15% of 21-d PR was on average $5.2 and decreased gradually until 20% 21-d PR. Different figures have been reported for the economic gain per percentage point of 21-d PR. The driving factors of this economic gain are the used input parameters (Cabrera, 2014). Thus, there is a considerable variation among different studies. Studies reported net return gain ($/cow per year per one percentage point of 21-d PR) as high as $18 (Galvão et al., 2013) and as low as $9 (Giordano et al., 2012), which gradually decreased with higher 21-d PR. The estimated economic gain ($/cow per year) of $5.2 in the current study compared to other studies could seem an underestimation. However, these values are highly dependent on the farm and herd simulated situation including input parameters, which are later discussed in the sensitivity analyses.

A great variation within each 21-d PR is observed (Figure 5.3). This high variation, which is mainly due to milk production differences among farms, offsets the improvement in 21-d PR. This higher variability would indicate that even farms with low 21-d PR can potentially have better annual net return than those with higher 21-d PR as long as their milk productivity is higher. However, the likelihood of achieving high net return for these farms is low, which should be considered within a risk preference decision-making framework.

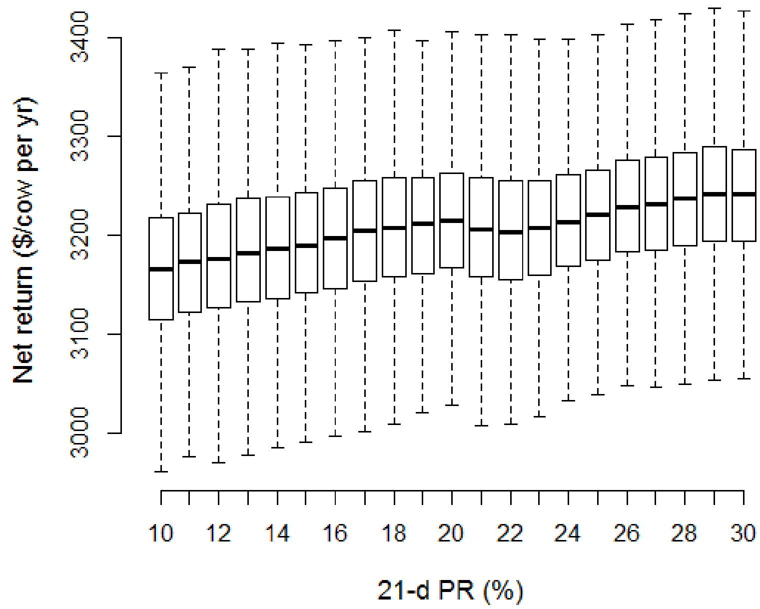

Figure 5.3. Net return ($/cow per year) variation across different 21-d pregnancy rates (21-d PR) from 2,000 replications after including involuntary culling, abortion, pregnancy rates, and milk production. Outliers were excluded.

Specific changes in the herd economics and dynamics at 5 different levels of 21-d PR (10, 15, 20, 25 and 30%) are portrayed in Table 5.3. As it would have been expected, the net return increased with higher 21-d PR (De Vries et al., 2010b; Fricke et al., 2010). However, the important factors affecting this higher net return were inconsistent among studies. Here the higher net return was due to the higher calf sales, lower culling rate, and lower reproductive costs. The effect of better reproductive performance on the income over feed costs was inconsistent among studies. Regarding milk production and sales, some studies (Hady et al., 1994; Fricke et al., 2010; Giordano et al., 2012) showed an increase in milk sales, whereas Galvão et al. (2013) showed that milk sales could decrease with better reproductive performance. In this study, the expected value of milk sales showed a reduction with increasing 21-d PR from 10 to 30% ($4285 vs. $4,184/cow per year). This could be explained to a large extent due to the lower proportion of lactating cows when higher 21-d PR, which results in lower milk production.

The difference of percentage of lactating cows when the model reached steady state between a 10% 21-d PR and 30% 21-d PR was 7.6 percentage points lower for the 30% 21-d PR. The lower milk sales for better reproductive performance was also observed in Galvão et al. (2013), and they also determined the main reason for it to be the lower proportion of lactating cows in higher 21-d PR. Nonetheless, other additional factors and their interactions could also play a role in this conundrum. The shape and level of milk lactation curves, especially persistency of the first lactation curve, can be of importance. (Cabrera, 2014) review on the subject speculated that the interaction of shape and peak of lactation curves with the changes in herd structure could result in less herd milk production even with higher 21-d PR when a large proportion of cows are in first lactation or earlier in all lactations when the lactation curves are highly persistent.

Table 5.3. Herd's economic and structure dynamics summary (average ± standard deviation) from 2,000 replications at 5 levels of 21-day pregnancy rate after including involuntary culling, abortion, pregnancy rate, and milk production levels as random variables into the model.

	21-day pregnancy rate (%)				
Parameters	10	15	20	25	30
Economics ($/cow per year)					
Net return	3166±74.5	3192±75.4	3215±73.2	3220±75.7	3241±74.5
Milk sales	4285±82.1	4228±83.0	4203±80.1	4191±82.4	4184±81.0
Feed costs	-820±7.3	-808±7.1	-802±6.7	-797±6.8	-794±6.7
Calf sales	51±0.7	64±0.6	71±0.3	76±0.2	80±0.2
Culling cost	-214±4.6	-178±3.3	-159±2.7	-147±2.5	-139±2.4
Reproductive cost	-137±14.2	-114±11.6	-99±12.5	-103±8.2	-90±7.3
Herd Dynamics					
Parity 1 (%)	49.0±0.7	37.2±0.4	30.7±0.3	27.1±0.3	24.8±0.3
Parity 2 (%)	25.0±0.4	24.5 ±0.2	23.0±0.2	21.7±0.2	20.8±0.2
Parity >= 3 (%)	26.0±1.0	38.3±0.95	46.3±0.95	51.2±1.05	54.4±1.3
Lactating (%)	86.6±0.18	83.4±0.12	81.4±0.08	80.0±0.06	79.0±0.04
DIM (day)	197.4±1.5	179±0.9	168±0.6	160±0.5	155±0.5
Open days (day)	155.0±2.3	141±1.4	130.0±0.9	120±0.7	112±0.6
Reproductive culling [1] (%)	11.0±0.68	7.3±0.4	5.0±0.2	4.03±0.14	3.5±0.1
Involuntary culling (%)	33.5±0.7	28.7±0.5	25.0±0.4	24.3±0.41	23.2±0.4
Mortality (%)	4.7±0.1	3.9±0.07	3.4±0.06	3.16±0.06	2.9±0.06

[1] Culling due to reproductive failure when DIM > 294 day and milk production < 23 kg/day

Results of this study, regarding the milk production, suggested that by improving reproductive performance of a given herd the overall milk production could decrease (Table 5.3), which could be an important point for dairy farms with quota on their milk production. Therefore, these farms could improve their reproductive performance, and use the higher net revenue generated from better reproductive performance, without exceeding their milk quota. It should be noted that productivity of an individual cow might increase in better reproductive performance, but the way it interacts with the herd structure (i.e., percentages of dry cows vs. lactating cow in the herd; depicted in Table 5.3 in percentage of lactating cows) keeps the overall herd's milk production at approximately same level.

Change in the 21-d PR was also the reason for significant changes in the herd structure (Table 5.3). For instance, at 10% 21-d PR the reproductive culling of approximately 11% is the main source of higher proportion of cows at the first lactation and therefore higher culling costs compared to 30% 21-d PR, which had only about 3.5% reproductive culling. This effect of different 21-d PR on the herd structure is in agreement with the daily Markov chain model in (Kalantari and Cabrera, 2012).

5.7.2. Sensitivity Analyses

Changes in the input parameters of milk price, replacement cost, milk threshold cut-off amount are summarized in Table 5.4. Among these parameters, milk price showed the largest impact on the overall net return. A 20% increase in milk price decreased the economic gain per extra 21-d PR (from $5.2 in base scenario average per unit gain in net return from 10% to 15% 21-d PR to $3.4, a 35% reduction) and 20% decrease in milk price showed a higher gain per extra percentage point of 21-d PR (from 5.2 in base scenario to $7.6, a 46% increase). At the other end of spectrum comparing 25% and 30% 21-d PR, shows a smaller increase. A 20%

increase in milk price decreased the economic gain ($/cow per year) per unit of 21-d PR from $4.2 to $3.8 (10% reduction). A 20% decrease in milk price increased the economic gain to $4.4 (from original $4.2). A similar trend, but not at the same scale, was also observed by Galvão et al. (2013), and they concluded that in periods of high milk price reproductive performance might not be as critical as periods of lower milk price (Galvão et al., 2013). Increasing the replacement cost by 20% increased the economic gain per unit of 21-d PR from 10% to 15% 21-d PR by 58% ($8.2 vs. $5.2). However, this gain at the 25% to 30% 21-d PR was much smaller and equal to $4.8. Compared to base of $4.2 this is just 14% improvement. A 20% decrease in replacement cost was similar to 20% increase but decreased the gain per unit from the base.

Table 5.4. Effect of changes in input parameters on net return ($/cow per year) from 2,000 replications across 5 different 21-d pregnancy rates

Parameter	Change (%)	21-d pregnancy rate (%)				
		10	15	20	25	30
Base scenario		3166±74.5	3192±75.4	3215±73.2	3220±75.7	3241±74.5
Milk price	+20	4031±90.8	4048±91.5	4064±90.6	4064±90.8	4083±91.7
	-20	2343±59.1	2381±59.5	2408±58.9	2414±59	2436±59.6
Replacement cost	+20	3076±73.7	3117±74.5	3146±74	3154±74.3	3178±75.2
	-20	3255±75.3	3270±75.6	3283±74.6	3282±74.6	3299±75.4
Milk cut-off threshold	-50	3082±76.9	3137±76.6	3181±75.1	3197±74.9	3225±75.6
Milk cut-off threshold + Replacement cost	-50 +20	2977±77	3050±76.6	3105±75.1	3129±74.9	3162±75.6
Milk cut-off threshold + Replacement cost	-50 -20	3189±76.9	3224±76.6	3256±75.2	3265±74.9	3288±75.6

Reducing cut-off milk threshold by 50% increased the average net return for each 21-d PR compared to the base scenario. Among all the changed parameters in Table 5.4, reducing the cut-off milk threshold had the highest impact on the extra gain per unit of change in 21-d PR. The average economic gain ($/cow per year) per unit of 21-d PR was $11 (more than twice) from 10% to 15% 21-d PR (compared to $5.2 in base scenario). From 25% to 30% the economic gain per 21-d PR unit was much lower ($6.6), but still considerably higher than the base scenario

($4.2). Different studies have used different criteria for culling non-pregnant cows. For example, Galvão et al. (2013) culled open cows whenever the daily generated revenue by the cow fell below their cost, or when they reached the cut-off DIM of 450. Authors of this study used the combination of cut-off DIM of 300 and milk threshold of 23 kg (Giordano et al., 2012). Selection of the culling criteria for non-pregnant cows seems to be highly dependent on the farms' policy, milk production level, and availability of better heifers. By decreasing the replacement cost (-20%) at the same time of reducing the cut-off milk threshold (-50%) resulted in the highest economic gain ($14.6/cow per year). This shows that the economic return is greatly affected by the input parameters and the gain would be considerably different from herd to herd and from region to region.

5.7.3. Implications for Farm Decision-making

In order to provide the overall distribution of net return of 5 levels of 21-d PR in Table 5.3, Figure 5.4 is provided. Cumulative density functions of these 5 pregnancy rates are displayed in Figure 5.4. There is a wavy shape in the curves due to the 15 milk classes and their distributions. This cumulative density distribution is not available in the standard Markov chain model and could be informative to farmers in decision-making processes. Decision makers can use these distributions to establish a target net return and calculate the probability of reaching such target value given their current 21-d PR. For example, Figure 5.4 can be used to find the probability of having a net return of at least $3,207/cow per year (average net return across 10, 15, 20, 25%, and 30% 21-d PR) for different 21-d PR. The obtained probability for 10 to 30% 21-d PR with 5% interval was 30%, 41%, 54%, 59%, and 65% respectively. This information alongside the risk preference of farmers can be used to make better informed decisions according to farmers' risk preferences. In the case of farms with 10% 21-d PR, 30% is the chance of having a net return above $3,207/cow per year. This probability could be useful guide for farmers' management strategies. It would tell them that to reach a target value of for example $3,207/cow per year, decision makers need to put a higher emphasis on improving farm's reproductive management and less emphasis on other aspects of management (such as culling, abortion and milk production), because the uncertainty of these other aspects are already being accounted. On the other hand, farms with 30% 21-d PR have 65% chance of having > $3,207/cow per year. Decision makers of these farms can sustain the current reproductive management and work on other management practices (such as culling, abortion and milk production), because there is a higher chance of getting higher net return (up to $3,467/cow per year) by improving those parameters.

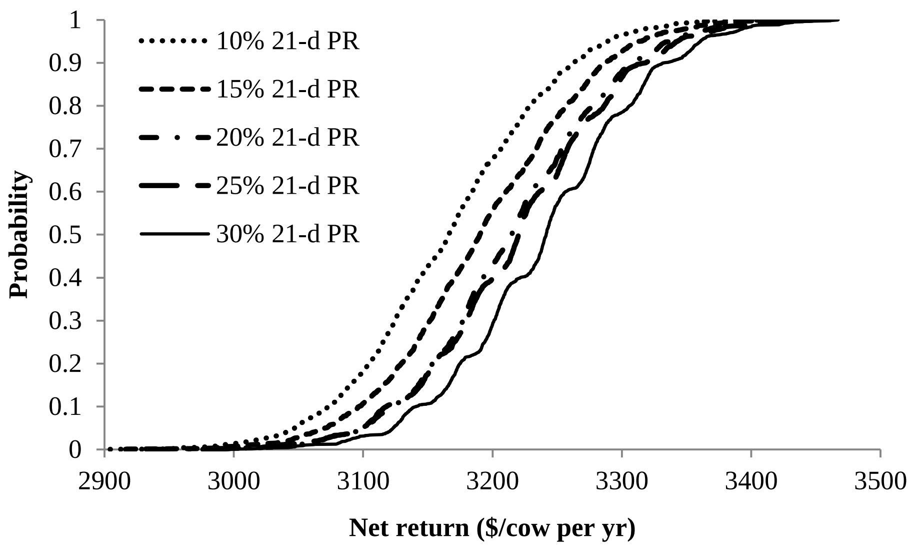

Figure 5.4. Cumulative density functions of expected net return ($/cow per year) for 5 different 21-d pregnancy rates (21-d PR) when all stochastic parameters (involuntary culling, abortion, pregnancy rates, and milk production) were included into the model.

5.7.4. Limitations

Like any other simulation and modeling research, this study has some limitations. Some diseases (such as mastitis, laminitis) play an important role in determining profitability of farms and also affect both milk production and reproductive performance. Therefore, for better economic evaluation, diseases and their interaction with reproductive performance and milk production need to be considered in future developments. In addition, this study avoided the possible antagonistic (agonistic) association between milk production levels and reproductive performance due to lack of conclusive results in the literature (Leblanc, 2010; Bello et al., 2012). This association is different at farm and cow level and further study is required to capture the essence of relationships between reproductive performance and milk production levels (Bello et al., 2012). Moreover, in this study, the potential genetic progress was not included and therefore the results presented here- especially those for higher reproductive performance- are probably underestimated. Finally, it should be mentioned that results in modeling studies, as the one presented here, are highly dependent on the input parameters and the underlying assumptions of the model, thus the economic gain and values presented herein are applicable to the situation of the present study only. Nonetheless, the modeling framework developed in this study could be useful in assisting research and still help on reproductive management on-farm decision making.

5.8. CONCLUSIONS

The model presented here demonstrated a simple way of including stochasticity into a Markov chain model to introduce uncertainty around transition probabilities. This helped to provide results for different reproductive performance under farms variable conditions and reducing the need for additional sensitivity analyses on transition probabilities. The modeling framework could serve as a guideline for future studies. An overall increase in the net return (US$/cow per year) from 10% to 30% 21-d PR was observed. However, within each level of 21-d PR, a considerable variation was also observed. Variability was mainly due to milk production levels, and it was found that even lower reproduction performance could attain a good net return as long as they have a high milk production level. However, the likelihood of having a high net return with low 21-d PR is low. An overall increase in net return across 21-d PR was mainly due to the lower reproductive and culling cost and a higher calf value. The resulted cumulative distribution of the net return across different 21-d PR could be informative to decision makers to guide them towards better informed decisions given their current reproductive performance and their risk preferences. Sensitivity analyses demonstrated that the economic return associated with reproductive performance is greatly affected by the input parameters and therefore herd-specific evaluations are critical.

5.9. ACKNOWLEDGMENTS

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Chapter 6

Economic Impacts of Nutritional Grouping in Dairy Herds

Section 1

A modeling approach to evaluate nutritional grouping of dairy cattle

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6.1. ABSTRACT

This manuscript is the first part of 2 companion studies designed to evaluate the economic effects of nutritional grouping in commercial dairy herds. This first part deals with model development and its verification and validation. The second manuscript analyzes the economic effects of nutritional grouping. A terminating dynamic stochastic Monte Carlo simulation model using next-event scheduling approach was developed to represent individual cows in herds and follow them on a daily basis. The model was initialized with characteristics of actual records of individual cows available from input herds. Thereafter, cow characteristics were updated daily based on stochastic events of involuntary culling, abortion, and pregnancy, and deterministic functions of DMI, NE_L and MP requirements. On a monthly basis, cows were ranked based on their NE_L and MP concentration requirements and assigned to the same or a different nutritional group. The developed model was verified to confirm that it performs as expected. Furthermore, the model was validated by comparing its projections against actual data regarding involuntary culling, BW, BCS, and NE_L and MP concentration of the offered diet. We concluded the model is functional to be used to analyze the economic effects of nutritional grouping of dairy herds.

Key words: dynamic simulation, nutritional grouping, feed efficiency

6.2. INTRODUCTION

A major constraint to enhanced feed efficiency on dairy farms is the lack of nutritional grouping (Vandehaar, 2011). Field experimentation might not be practical or feasible to study the economic value of applying nutritional grouping because it could be very expensive, time consuming, complex, highly variable, and difficult to control. Furthermore, the results might not be generalizable to other herds because net returns of nutritional grouping are herd specific, sensitive to herd structure and size, and dependent on market circumstances. Under these conditions, simulation studies can be of assistance and valuable to gain a better understanding of the system behavior and the economic effects of nutritional grouping. Simulation can provide the opportunity to explore complex relationships between input factors that are difficult or impossible to study in the field (Kristensen et al., 2008).

Indeed, simulation models have been previously developed and used to quantify the economic and environmental impact of grouping on a nutritional basis for the lactating cows in a herd (Williams and Oltenacu, 1992; Pecsok et al., 1992; Østergaard et al., 1996; St-Pierre and Thraen, 1999; Cabrera et al., 2012). In the current study, we advance the modeling of nutritional grouping by using a special type of stochastic Monte Carlo simulation approach, next-event scheduling (De Vries, 2001), which can be used to model individual cows daily for a limited period of time. With this structure, the daily energy balance of each cow in the herd can be computed and followed for long periods of time (i.e., a year). To the best of the authors' knowledge, no previous study has included daily accounting of energy balance in a stochastic simulation model.

The next-event scheduling approach dynamically and stochastically predicts the herd structure based on key input parameters such as culling risk, pregnancy, and abortion. Also, limited time or terminating simulations as the one developed here, predict the behavior of the system in the short-run. In this method, the system starts at a specific initial point via an initialization using snapshot of a herd (i.e., current herd structure), and then an event (e.g., parturition) or running time (e.g., a year) determines the end point of the simulation (Banks et al., 2009). Initialization based on a herd's data enables the model to provide a projection for an input herd over a course of defined time, which is dependent on the current herd structure and characteristics of the input herd. Thus, this modeling approach is useful in evaluating the herd-specific economics of nutritional grouping strategies.

In addition, the introduction of cow-individual and daily body energy to estimate BW and BCS within the proposed simulation modeling approach is unique and innovative. Because nutritional grouping strategy impacts the energy concentration of the DM offered and consumed, our proposed grouping model updates body energy of the cows on a daily basis, by rectifying the predicted amount of energy consumed and expended, depending on current BW, BCS, DMI, and dietary energy concentration. Individual cow nutrient requirements in the proposed model are deterministic, but these pertain to a cow that is following stochastic events.

Studies have used different methods to determine the NE_L and CP concentrations of a diet for a group of cows, but in general all used the average milk production of a group as the basis for calculating lead factors. These methods are, for example, the use of the 83rd percentile in each group (Stallings and McGilliard, 1984) or the use of differentiated levels according to a number of groups (Stallings, 2011). The 83rd percentile method proposes the formulation of the diet based on the 83rd percentile cow milk production in the group. This, approximately, corresponds to formulating the diet to 1.31 standard deviations above the average milk production for the group (1.31 lead factor; Stallings and McGilliard, 1984) assuming milk production in the group follows a normal distribution (St-Pierre and Thraen, 1999). The other method proposes formulation of diets based on group's milk production and number of nutritional groups such as diets for 30, 20, and 10% above the group's average milk production for nutritional groups 1, 2, or 3, respectively (Stallings, 2011). In the current study, we go beyond milk production as proxy by using individual cow's overall NE_L and MP requirements to formulate more precise group diet nutrient concentrations.

Thus, the objectives of this study were to 1) describe the development process of the dynamic stochastic simulation model to simulate adult dairy cows for the specific purpose of evaluating the nutritional grouping, and 2) to verify and validate the results of the model. An application of the model to evaluate the economic effects of nutritional grouping in commercial dairy herds is presented in a companion paper (Chapter 6 Section 2).

6.3. MATERIALS AND METHODS

6.3.1. Simulation Approach

A dynamic stochastic Monte Carlo simulation was developed to model cows after first parturition in a dairy herd. The next-event scheduling approach (De Vries, 2001) was used to simulate individual cows and stochastic events that could happen to cows during each reproductive cycle. First, a dataset of individual cows in a herd and their current status were loaded (i.e., lactation number, day postpartum, reproductive status, etc.). Then, a list of possible stochastic events was scheduled for each cow at the beginning of the simulation and the list was renewed after starting their next lactation. These events included: involuntary culling, death, pregnancy, abortion, dry-off, and parturition. For each event, a 2-step process was followed: 1) determining the binary outcome of the event (it happens or not during the cow's current lactation) and if it happens 2) the day of the occurrence (schedule).

The first step consists of generating a random number from a U~(0, 1) (De Vries, 2001), which is used to determine the outcome of an event by comparing it to a testing threshold. For example, if the probability of a cow being culled in the first lactation is 0.17, a random number ≤

0.17 indicates that the cow would be culled during that lactation. The second step consists of generating another random number, from an appropriate distribution, to schedule the day the event would occur. For example, in the case of culling, the appropriate distribution could be obtained by creating a cumulative distribution function (**CDF**) from the historical data of involuntary culling. Following the example of culling, data adapted from Pinedo et al. (2010; Table 6.1) was used as the distribution of involuntary culling. Details of this process are described in the following section. The process of scheduling the event was the same for all the events. When the underlying distribution of time of the event was known from previous studies, those standard distributions were used (Table 6.1). However, when the underlying distribution was unknown, an empirical distribution was used to approximate the time of the events.

After scheduling all the events, every cow is evaluated on a daily basis to check if the cow matches to any specific event on a given day postpartum. If the event matches with the current day postpartum, then the cow status is updated to reflect the latest changes in the cow's attributes. For example, if a cow is 90 d postpartum and the cow was scheduled to be pregnant on that day, then the cow information is updated as pregnant. In the cases of culling and mortality events, replacement (with a first calving heifer, pregnant ready to calve) is assumed to occur the next day to maintain the herd size constant over the course of the simulation.

It should be noted that a given cow is affected by the first matched event with the current day postpartum. Therefore, if there were any other events scheduled for the given cow, those would remain on hold until the scheduled day postpartum, which might not happen depending on what occurred on the previous events. For example, if the cow was scheduled for culling at 400 d postpartum, but calved at 350 d postpartum, then the culling would not occur, the cow would move to the next lactation when a new set of events would be scheduled for that cow.

Table 6.1. Thresholds and distributions for scheduling cow life events on the Monte Carlo model

Event	Threshold (%)	Event Distribution	Mean	SD	Min	Max	Source
Death		Empirical					
Parity 1	3.9						Pinedo et al. (2010)
Parity 2	5.6						Pinedo et al. (2010)
Parity 3	8.5						Pinedo et al. (2010)
Parity ≥ 4	11.7						Pinedo et al. (2010)
Involuntary culling							
Parity 1	16.9						Pinedo et al. (2010)
Parity 2	23.3						Pinedo et al. (2010)
Parity 3	30.1						Pinedo et al. (2010)
Parity ≥ 4	40.8						Pinedo et al. (2010)
Involuntary culling reason[2]							
Feet and Legs	16.33	Empirical					Pinedo et al. (2010)
Injury	28.83	Empirical					
Mastitis	24.39	Empirical					
Disease	13.91	Empirical					
Udder	6.45	Empirical					
Unknown	10.09	Empirical					
Reproduction							
First ovulation		Lognormal	19	11	2		De Vries (2001)
Heat detection	50	Uniform			0	1	
Conception rate	40	Uniform			0	1	
Estrus length		Normal	21	4	12	30	Oltenaco et al. (1980)
Dry days		Uniform			45	55	Allore et al. (1998)
Gestation length		Normal	278	6	266	290	Oltenaco et al. (1980)
Pregnancy loss	8	Empirical					
Body weight		Triangular	700		550	1000	Authors
BCS[1]							
Heifer		Triangular	3.3		3.5	3.7	
d postpartum =1		Triangular	3.0		3.5	4.0	
d postpartum ≤70		Triangular	2.0		2.5	3.0	
d postpartum <250		Triangular	2.5		3.0	3.5	
d postpartum ≥250		Triangular	3.0		3.5	4.0	

[1] Body condition score distribution was assumed to be dependent on stage of lactation.

6.3.2. Stochastic Events

Scheduling of culling and reproduction events are performed at the beginning of the simulation and after a cow starts a new lactation. At the beginning of the simulation, the event scheduler takes into consideration the current days postpartum of the initialized cows and only schedules possible remaining events. Events define the herd structure and are the cornerstone of the next-event scheduling simulations, which are described in more detail below.

6.3.2.1. *Involuntary Culling and Death*

Whether or not a cow is going to finish a given lactation was based on the annualized live culling and death rate from Pinedo et al. (2010; Table 6.1). For example, the threshold for death rate of the first lactation cows was set at 3.9%, which means that 3.9% of first lactation cows were scheduled for the death event. After knowing if a given cow would leave the herd, the reason for it needed to be known. Also, data from Pinedo et al. (2010) was used to get the CDF for different reasons of involuntary culling. Inclusion of different reasons helped capture the fact that based on the nature of diseases the culling could happen at different days postpartum and affect differently the herd, both economically and structurally. Thresholds for the reasons of involuntary culling were obtained by excluding the low production and reproduction reasons (voluntary culling) from the data in Pinedo et al. (2010). Then, all the involuntary culling percentages were reset to 100%. These modified percentages are shown in Table 6.1. Then, random U~(0,1) were generated and compared with these thresholds to determine the reason for culling. The last step was to schedule the time of the event. As an illustration, distributions for mastitis and death events are selected in Figure 6.1. The inverse-transform method (Banks et al., 2009) was used to generate random variates from these empirical distributions. The first stage in this process was to find the CDF of all the events, which is illustrated in Figure 6.1 Panel B. It should be noted that the distributions in Figure 6.1 Panel B are discrete, but shown continuously for simplicity. The dots in Figure 6.1 Panel B are the data available, and therefore an interpolation was needed to find an approximate event time between the intervals. In a second stage, lines connecting the dots (Figure 6.1 Panel B) were used to calculate the inverse of the empirical CDF and from it, the slope between 2 successive points. The slope was calculated based on formula from Banks et al. (2009), as follow:

$$a_i = \frac{x_i - x_{i-1}}{c_i - c_{i-1}}, \qquad [1]$$

Where c_i is the cumulative probability of the i^{th} interval and x_i is the i^{th} interval of the corresponding x (days postpartum in Figure 6.1 Panel B). Finally, the inverse was calculated using:

$$X = x_{i-1} + a_i \times (R - c_{i-1}), \qquad [2]$$

Where X is the random variate generated from the target empirical distribution, and R is a random number U~(0, 1). For example in the case of mastitis with an R=0.48, which is between CDF of c_6=0.432 and c_7=0.562 and the corresponding x (the time of the occurrence of mastitis) are x_6=135 and x_7=180, using Eq. 1 results in a_7=346. Eq. 2 gives the random variate, which after rounding to the closest integer is equal to 152 (X_1=135 + 346 × (0.48 - 0.432)). Thus, for an R=0.48, the time of culling due to mastitis is 152 d postpartum. The same process for the death event results in 87 d postpartum. Adding replications generated distributions of the events.

Besides involuntary culling, reproductive culling of non-pregnant cows was included in the model. Non-pregnant cows with days postpartum > threshold (e.g., 300 d) were marked as do-not-breed and were culled (reproductive failure) whenever their milk production dropped < milk threshold (e.g., 24 kg/d). The cut-off days postpartum was different among farms and was implemented as an input parameter. Cows reaching the end of the 10th lactation were set to be voluntarily culled whenever their milk production dropped below the threshold.

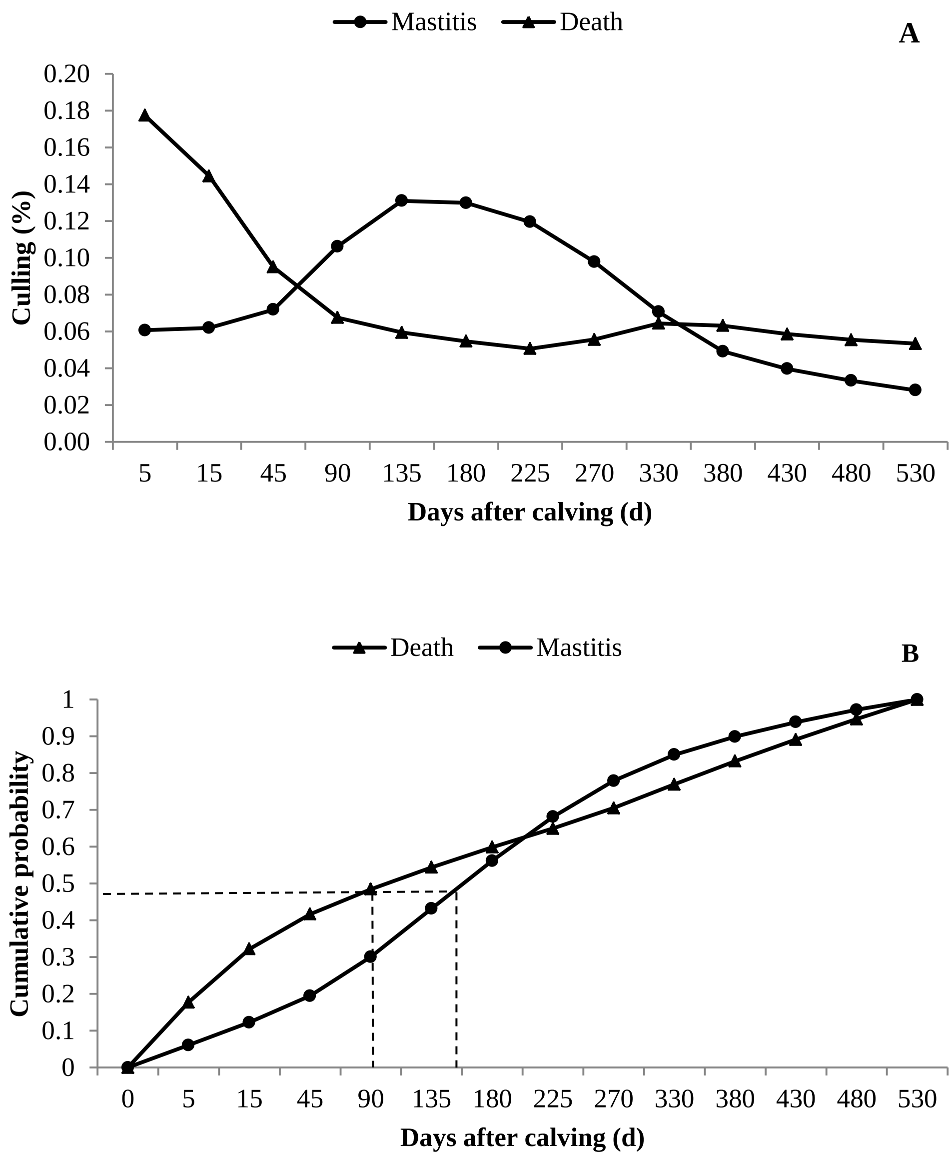

Figure 6.1. Panel A: Distribution of culling (%) for mastitis and death events through days after calving adapted from Pinedo et al. (2010). Panel B: Cumulative distributions function of mastitis and death for the process of generating random variates from empirical distribution.

The first occurrence of postpartum ovulation was modeled using a lognormal distribution (De Vries, 2001). Cows with days postpartum ≥ defined voluntary waiting period of 50 d were observed for estrus and had risks for breeding and conception. The estrus cycle length was determined from an N~(21, 4^2) distribution, truncated at 12 and 30 d (Table 6.1). Cows detected in estrus (estrus detection) and conceived (conception rate) were marked as pregnant. These rates were herd specific and are summarized in Table 6.2 for 5 studied commercial dairy herds. The process of estrus detection and conception was repeated with stochastic intervals based on variable estrus cycle lengths until a cow became pregnant or reached the cut-off days postpartum (e.g., 300 d). Parturition was scheduled according to the gestation length following N~(278, 6^2) truncated at 266 and 290 days (Oltenacu et al., 1980), after the date of pregnancy was determined. Next, the dry-off date for pregnant cows was scheduled following a U~(45, 55) (Allore et al., 1998b) by back-calculation from the parturition date. The impact of extra days open on the dry period was also taken into account. An extra day was added to the dry period for each 2 open days after 120 d open (Allore et al., 1998b). A herd specific risk of pregnancy loss (Table 6.2) was considered from 30 d to term with an empirical distribution determining the day of occurrence (Giordano et al., 2012).

Table 6.2. Input dairy herd characteristics and structure at the beginning of the simulation

	Herd Size (Lactating + Dry)				
Characteristics	331	570	727	787	1,460
Average Herd ME305[1] (kg/cow per yr)	13,348	16,140	13,897	12,884	14,188
1st Lactation (%)	38	43	39	39	45
Average days in milk[2] (d)	193	169	181	165	174
Average days in Pregnancy (d)	134	140	141	133	157
Average lactation number	2.03	1.99	2.29	2.21	2.02
21-d Pregnancy Rate[3] (%)	17	18	19	19	18
Conception Rate[3] (%)	35	32	36	37	40
Estrus Detection[3] (%)	49	57	51	51	45
Culling[3] (%/yr)	35	32	36	37	40
Abortion[3] (%/gestation)	16	7	11	11	7
Cow BW available	Yes	No	Yes	No	No
Cow BCS available	No	No	No	No	No

[1] 305 d mature equivalent milk production

[2] Average days in lactation

[3] As defined and calculated in DairyComp305 (Valley Agricultural Software, Tulare, CA)

6.3.3. Variation Control

Stochastic simulation models usually need a variance reduction technique to control variation of the generated random numbers to provide precise and relevant information regarding the problem studied. This variation control is especially useful when the model is large and takes considerable computational time to solve (Hillier and Lieberman, 1986) as it was the case in this study. Therefore, a synchronized common random numbers method was used (Banks et al., 2009) to control the variation due to differences in generated random numbers. In this method, the same stream of random numbers (common random numbers) is used for the same purposes (synchronized) among different scenarios of the model. This method diminished the need for a large number of replications to reduce the standard deviation of outcomes. The synchronization was done in 2 steps. First, a 6 digit integer number, chosen arbitrarily, was set as the common seed for all herds beforehand. This seed was shared by all the herds at the beginning of every simulation. However, each replication of the model had a unique ID by adding a sequential replications number to this shared seed. Thus, each replication of a given herd had a unique seed for generating random numbers, and the seeds were different among replications. Second, the common seed generated in the first step was added to the ID of each cow in the herd in each replication. Thus, each cow in the herd was running on the same stream of random numbers within each replication, but on different streams among replications. One more step was necessary for scheduling involuntary culling, which created independent random number streams for each stochastic culling event. Culling schedule events required first checking if the cow was going to be culled, then determining the culling reason, and finally scheduling the date for the event. Each one of these random numbers was independent of each other.

6.4. Modeling Framework

An object-oriented design was used to assist in the design process and implementation of the Monte Carlo simulation computer programming. An object-oriented design is a well suited approach to design biological systems, and is considered the most straightforward technique for converting data from real world objects into a computer based model. This is mainly due to the fact that one can easily draw direct relations among components of a herd and the data needed to represent the given object in the computer model (Jørgensen and Kristensen, 1995; Sequeira et al., 1997). For instance, cow, group, and TMR can all be described in the model, virtually, as entities, which interact with possible stochastic events. Object-oriented design principle was followed in 2-step processes to build the model. First, all the important components of the herd related to the nutritional grouping were identified. This step dealt with how the system works focusing on interactions among cows within a herd and identifying important attributes of the cows and the herd, which are related to the nutritional grouping. This step's main goal was to recognize important objects in a real herd that could play an important role in a nutritional grouping and define them as objects: i.e., herds, cows, groups, and TMR. Detailed descriptions of these objects and their relevant attributes follow in the next section.

Second, object-oriented programming, a method of implementation of a conceptual model to a computer program, was used to build the model (Shaffer et al., 2000). Java 1.7, a concurrent and a general object-oriented programming language, was used to implement the model. The model benefited from concurrent characteristics of Java to run replications of a given herd simultaneously, which was especially useful in decreasing the running time.

The model was run for 1,000 replications and both herd dynamics and economic outputs were stored as comma separated files for further aggregation to report key figures and statistics. These replications were considered different instances of the herd being simulated. Results were accompanied by standard deviations for each herd statistic to include the inherent uncertainty in the results.

6.5. Objects

Objects relevant to the nutritional grouping are herd, cows, groups, and TMR (Figure 6.2). The process of acquiring data from the input herds and the data flow inside the program is illustrated in Figure 6.2, Panel A. Other objects are designed to make the computer model more manageable and to allow for future modifications. However, because they are not directly relevant to the nutritional grouping of cows, these are not discussed herein. For example, the Initializer in Figure 6.2, Panel A was used to instantiate the herd object and store some of the input constants that are shared among herds, such as the simulation period (number of days to simulate the herds) and prices (milk price and TMR related prices). The following sections describe attributes related to the other objects named in Figure 6.2, Panel A.

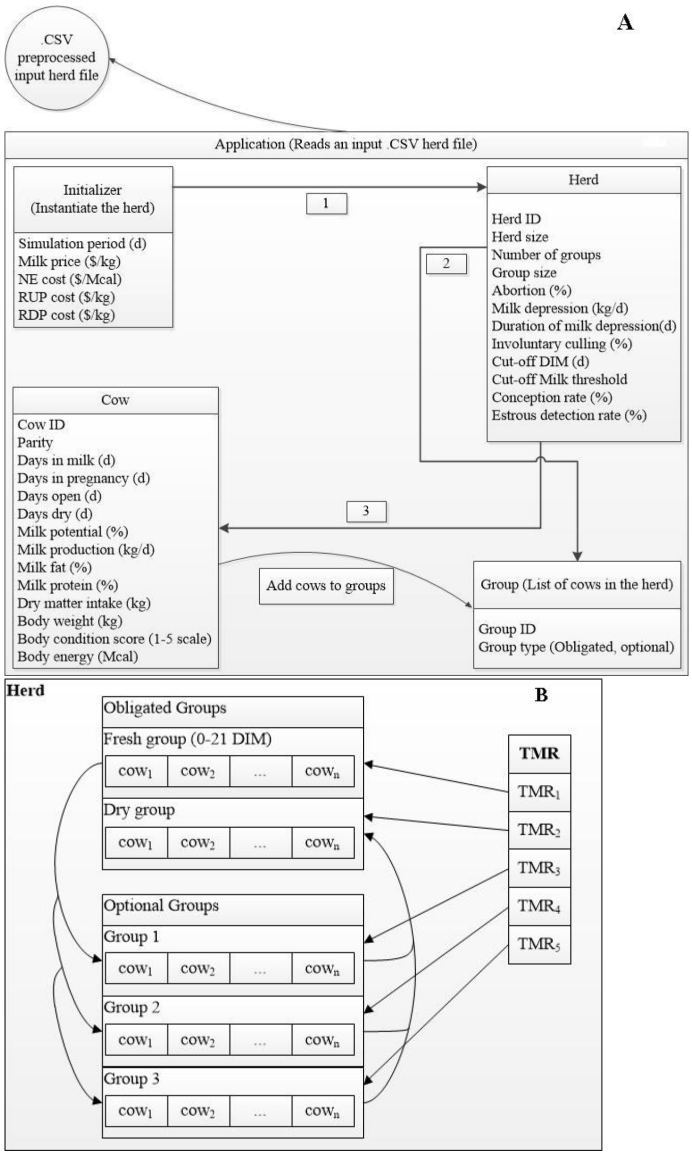

Figure 6.2. Diagram flow of simulation modeling framework. Panel A: The main application reads a preprocessed input file and then *Initializer* (1) is used to instantiate the herd object. Following completion of this process *Groups* are created (2), and cows are being initialized based on the input cows from a herd (3). Finally these cows are added to the appropriate group (obligated or optional). Panel B is a schematic representation of a herd with 3 optional groups for lactating cows. At the beginning of the simulation (day 0) and at the end of each month the cows are ranked based on their nutritional requirements, are regrouped among the optional groups, and fed with a TMR formulated based on the group requirements. Depending on their state, cows move into a dry group or fresh group.

6.5.1. Herd

The underlying object in the model is the herd. All the data regarding an input herd were considered deterministic and stored in the herd object, which were used throughout the simulation as constants. Figure 6.2, Panel A summarizes important attributes used to define a herd in the model. These attributes were initialized based on the real herd characteristics. Most of the herd attributes are self-explanatory and were used to instantiate the cows in the herd, which will be discussed later. The developed model was parameterized based on the average values available from each herd separately. Herd characteristics for 5 commercial herds are summarized in Table 6.2.

6.5.2. Cow

A cow is described by a series of state variables (attributes). These attributes are summarized in Figure 6.2, Panel A. After initializing a herd, cows were instantiated and added to the herd based on the input file. An input file, a snapshot from current adult cows (lactating and dry cows) of a commercial herd, was read and stored in a database of all the cows. Variables included in the input file were: cow id, parity, days postpartum, days in pregnancy (**DIP**), fat percentage, protein percentage, 305 mature equivalents (**ME 305**), BW (if available), BCS (if available), and predicted producing ability (**PPA**; producing ability of a cow in future lactations, if available).

Cow attributes. The following sections describe the methods used to calculate and update different attributes of an individual cow on a given lactation and days postpartum throughout the simulation. Some of these attributes were held constant for the lifetime of a cow (e.g., PPA), and some were changed based on the lactation and days postpartum (e.g., lactation curves). Thresholds and distributions used for determining the outcomes of an event and its corresponding time of occurrence are summarized in Table 6.1 and the process is described in the simulation approach.

Milk, fat and protein production. Cows in the herd were categorized based on their PPA or, in its absence, their ME305. The PPA, which is calculated by the US Department of Agriculture as a part of the genetic evaluation program, predicts how much milk a cow can produce in subsequent lactations compared to herd mates. Cows were classified based on their PPA to a reference milk production of 11,828 kg (26,077 lb), which is the annual milk production of an average US Holstein born in 2005 according to the records from the US Department of Agriculture. Thus, every cow was classified based on their future capability of production relative to this reference by scaling up or down their production curves. In the absence of PPA, level of milk production for a cow was estimated based on the cow's projected ME305 relative to the average ME305 of a herd. This attribute is named as the milk potential in Figure 6.2, Panel A. This value captured the level of milk production of a cow compared to herd mates and was held constant over the simulation period.

After setting a relative level of production for each cow, the incomplete gamma function (Wood, 1967a) was used to calculate daily milk production of a cow. First, second, and third and later lactations used different prediction curves of milk production. These parameters were estimated based on the reference milk curves obtained from dairy herd improvement records (AgSource Cooperative Services, 2013). All the herds in the study shared the same shape of lactation curves. This was done to decrease the dominating effect of milk production on the outcomes of the model. However, every single cow had different milk production, because its production was relative to its herd mates and the herd average. Additionally, a random dependent milk production error was added based on Allore et al. (1998) and De Vries (2001) to capture the possible daily fluctuation of milk production for each cow:

$$M_t = 0.9896 \times M_{t-1} + e_t \ , \qquad [3]$$

Where M_t is a random dependent milk production error and e_t is the random independent residual on t days in lactation and follows $e_t \sim N(0, 0.45^2)$. Milk production was also adjusted to decrease by constant factors of 5, 10 and 15% by month of pregnancy 5, 6, and 7, respectively (De Vries, 2004). Modified Woods curves for better fit as described in De Vries (2001) were used to find the milk fat and protein percentages during lactations. Milk production level of replacement springer heifers was assumed to follow a N~(100, 10.02) based on (van Arendonk, 1985b).

Body weight and BCS. The initial BW and BCS for an individual cow could come from dairy herd records or from distributions. For those herds without BW or BCS records, a triangular distribution was used to generate the initial BW and BCS at the beginning of the simulation. Triangular distribution for BW was based on 794 available BW records from 2 Holstein farms. Based on these data the minimum BW was set at 550, the maximum at 1,000, and the most likely BW to 700 kg- T~(550, 700, 1000). These calculated BW were used as the mature BW for each cow in the herd and factors of 0.82 and 0.92 were used to set the current BW for the first and second lactation cows, respectively (NRC, 2001). Then, the difference between current BW and mature BW was used to estimate the average daily gain (**ADG**) needed for the first and second lactation cows to reach their mature BW, as follow (NRC, 2001):

First lactation:

$$\text{ADG (kg/d)} = ((\text{MW (kg)} \times 0.92) - \text{BW (kg)}) / 300, \qquad [4]$$

Second Lactation:

$$ADG\ (kg/d) = (MW\ (kg) - BW\ (kg)) / 300, \qquad [5]$$

where MW is the mature BW and BW is the current BW. It was assumed that cows use the consumed energy for growth after passing the average negative energy balance period at the beginning of the lactation (65 d postpartum). Thus, 65 d were deducted from an average calving interval of 365 d to uniformly distribute the net weight gain (growth) over a current lactation. Third and later lactation cows were assumed to be at their MW. For initializing BCS, different triangular distributions were used based on the cow's days postpartum. For primiparous cows entering the herd, a less variable distribution of T~(3.3,3.5,3.7) was used. For other cows, the expected BCS based on their days postpartum was used. Distribution parameters used for BCS and the range of days postpartum are summarized in Table 6.1.

After setting the initial values for BW and BCS, these values were used to find the total body energy (**BE**) of each cow in the herd based on NRC (2001; p. 24) equations. The estimated BE enabled the model to keep track of excess or deficiency in energy intake of an individual cow based on the energy provided by the diet on a daily basis. Moreover, this BE was used to predict the BW and BCS of each cow in the herd over time.

To capture the relationship between BE and standard BW (standardized at BCS 3.0), 13 samples of BE for cows at different sizes and BCS were generated (from 400 to 1000 kg with 50 kg intervals and over 5 BCS scores with 0.01 increments) based on NRC (2001) equations. These samples were used in a 2-step regression analysis as follows: first, a third order polynomial regression was used to predict BW from the BE of a cow on available 13 samples of generated body weights, as follows:

$$BW(kg) = \beta_0 + \beta_1 BE^3 + \beta_2 BE^2 + \beta_3 BE, \quad [6]$$

Applying Eq. 6 on 13 samples available provided 13 different regression lines. Each parameter in equation 6 was examined to capture the trend in the parameter changes across BW. The linear term (β_3) was constant across BW, whereas the intercept changed linearly, therefore these coefficients were found for a given BW and BE of a cow. Obviously, the high order terms (β_1, β_2) were curvilinear. Second, using this information a fifth order polynomial regression was used to capture the essence of changes of parameters (β_1, β_2) in Eq. 6:

$$\beta_{1,2} = \alpha_0 + \alpha_1 BW^5 + \alpha_2 BW^4 + \alpha_3 BW^3 + \alpha_4 BW^2 + \alpha_5 BW, \quad [7]$$

Eq. 7 was used to predict two terms (β_1, β_2) of Eq. 6 and the intercept and linear terms were calculated and applied for a given standard BW and BE of a cow to predict a new BW for a cow based on Eq. 6. The exact same process was used to predict the BCS of cows based on their BE. Use of higher order polynomial assured a perfect prediction of BW and BCS from BE (R^2 =1.0). The described method was used to capture all the changes in BE of a cow based on the consumed energy and predict the resulting synchronized BW and BCS. This procedure tracked the balance of energy throughout the life of each cow in the herd on a daily basis.

To control the BCS range, some lower and upper bounds were set. The lower bound was set at 2.0. Cows that would drop below this BCS were assumed to stay at this BCS = 2.0, but at a decreased milk production (New daily milk (kg/d) = daily milk (kg/d) – (deficient NE_L (Mcal/d) / NE_L in Milk (Mcal/kg))). Their milk production was deducted by an equal amount of energy deficiency. The upper bound was set at 4.5. Cows that would reach greater than this BCS were assumed to stay at this BCS = 4.5, but decreased their DMI (New DMI (kg/d) = DMI (kg/d) – (extra NE_L (kg/d) / dietary NE_L (Mcal/kg))).

Deterministic Parameters

Dry Matter Intake. Daily DMI was calculated using the NRC (2001; p. 4) equation, which is a function of BW and fat-corrected milk production with an adjustment for decreased DMI during the early postpartum period.

Cow Nutrient Requirements

Net Energy. Total net energy (NE) requirement of a cow was calculated by aggregating the requirements for maintenance (NE_M), milk production (NE_L) and growth (NE_G) based on NRC (2001) equations.

$$NE_M \text{ (Mcal/d)} = 0.079 \times BW^{0.75} \text{ (kg)} , \quad [8]$$

$$NE_L \text{ (Mcal/d)} = \text{MilkY (kg/d)} \times (0.0929 \times \text{Fat (\%)} + 0.0547 \times \text{Protein (\%)} + 0.192), \quad [9]$$

$$NE_G \text{ (Mcal/d)} = 0.0635 \times (EQEBW^{0.75} \text{ (kg)}) \times (EQEBG^{1.097} \text{ (kg)}), \quad [10]$$

where EQEBW is the size scaled empty BW, and EQEBG is the size scaled empty BW gain. The formula for calculation of these values can be found in NRC (2001; p. 320). Dietary NE concentration requirement (Mcal/kg DM) was calculated by summing up all the components from equations 8 to 10 and dividing the resulting sum by the DMI of each cow.

Metabolizable Protein. The total metabolizable protein (**MP**) requirement was also calculated by aggregating the MP requirements of maintenance (MP_M), milk production (MP_L) and growth (MP_G) based on NRC (2001) equations.

MP requirements for maintenance:

$$MP_M\ (g/d) = SR\ (g/d) + UR\ (g/d) + MFR\ (g/d) + ER\ (g/d), \quad [11]$$

Where SR is scurf protein requirement = ($0.3 \times BW^{0.6}$ (kg)), UR is urinary requirement = ($4.1 \times BW^{0.5}$ (kg)), MFR is metabolic fecal protein requirement = (DMI (kg) × 30 – 0.5 × ((MPBact / 0.8) – MPBact)), ER is MP required for endogenous protein = (0.4 × 11.8 × DMI (kg)) / 0.67, MPBact is the metabolizable protein supplied by the microbial protein (g/d) and it depends on total digestible nutrients for production (TDN_p (%) = ((NE_L (Mcal/kg) × 0.92) + 0.12) / 0.0245). The TDN_p was further used to find the microbial CP (g/d) = (DMI (kg) × TDN_p (%) × 130). Finally, a conversion factor of 0.64 was applied to convert Microbial CP to MPBact.

MP requirements of lactation:

$$MP_L\ (g/d) = ((MilkY\ (kg/d) \times MTP\ (\%)) / 0.67) \times 1000, \quad [12]$$

where MilkY is the milk production and MTP is the milk true protein (milk crude protein - 0.19).

MP requirement of growth (MP_G) was calculated in 2 steps: first, net protein for growth (NP_G) was calculated, and then, the efficiency of converting NP_G to MP_G was considered:

$$NP_G\ (g/d) = ADG\ (kg/d) \times (268 – (29.4 \times (NE_G\ (Mcal/d) / ADG\ (kg/d)))), \quad [13]$$

where ADG and NE_G are the same as above. The efficiency of converting NP_G to MP_G based on NRC (2001) was set at 0.28908 unless the equivalent shrunk body weight (EQSBW) ≤ 478 when the equation from NRC (2001) p. 320 was used. The MP_G was then found by dividing NP_G by the efficiency of conversion. Finally, MP was calculated by aggregating the results of

Equations 11 to 13. Furthermore, this MP was divided into rumen degradable protein (RDP) and rumen undegradable protein (RUP), which were calculated as follows:

$$\text{RDP (g/d)} = \text{DMI (kg)} \times \text{TDN}_p\ (\%) \times 153, \qquad [14]$$

$$\text{RUP (g/d)} = \text{MP (g/d)} - \text{MPBact (g/d)}, \qquad [15]$$

Nutrient Captured in Milk. Protein captured in milk for each herd was estimated by calculating the total N excreted in milk and dividing it by the total CP ingested in the diet.

$$\text{milkN} = (\text{MilkY (kg)} \times \text{milkPr (\%)} / 6.38) / (\text{DMI (kg)} \times \text{CP (\%)} / 6.25), \qquad [16]$$

where milkN is the percentage of nitrogen in milk (%); milkPr is the protein percentage in milk, and CP is the percentage of CP in the diet. The CP percentage was calculated by adding up RDP and RUP after considering 80% digestibility for RUP. Energy captured in milk was calculated based on the amount of energy available in the milk produced and dividing it by the total energy ingested from the diet.

6.5.3. Group

A group of cows share a common diet. The model distinguished 2 types of groups as follows: obligated and optional. Obligated groups consisted of a group for dry cows and another group for fresh cows. Obligated groups were considered standard in all herds and not part of the analysis of nutritional grouping. Figure 6.2, Panel B depicts a schematic representation of the groups and the flow of the cows among the groups. Cows in the fresh and dry groups were fed based on a constant NE_L of 1.7 and 1.28 Mcal/kg DM, respectively. For all other lactating cows NE_L was based on the group with a maximum NE_L concentration (Mcal/kg) of 1.79. The MP concentration of requirements (g/100 g DM) for the fresh cows was estimated based on the 83rd percentile of the requirements in this group of cows, and this value for dry cows was set at 7 g/100 g DM (approximately 1000 g/day recommended amount for an average mature cow). Cows in the fresh group were moved to an optional group after 21 d postpartum. Optional groups were used to test the effect of nutritional grouping on the overall income over feed costs (**IOFC**) and efficiency of the nutrients used in milk production by the cows. For consistency of analyses among herds, total lactating cows were grouped approximately in equal sizes in a predefined number of groups based on the herd original input herd size (Table 6.2).

6.5.3.1. *Grouping Strategy*

Clustering method was used to group the cows simultaneously based on their energy and protein concentration requirements following a modified version of the method introduced by McGilliard et al. (1983). In the current model, Euclidian distance between the standardized NE_L and MP was used to assess the similarity among the cows based on their NE_L and MP concentration requirements (CP and the squared of the Euclidian distance was used in the McGilliard et al. (1983) study). In this method after calculating the standardized NE_L and MP concentrations (μ=0, σ^2=1) for each cow in the optional groups they were ranked based on the calculated Euclidian distance (d(A,B)) among all the cows as follow:

$$\mathrm{d}\ (\mathrm{A,B}) = \sqrt{\left(SNNE_{L(A)} - SNNE_{L(B)}\right)^2 + \left(SNMP_{(A)} - SNMP_{(B)}\right)^2}, \qquad [17]$$

Where $SNNE_L$ is the standardized NE_L requirements concentration and SNMP is the standardized MP requirements concentration of the cows to be compared. This distance was calculated for all the cows in a herd with a seed cow, which was set to the cow requirements of > 10 standard deviation above the mean for both NE_L and MP (McGilliard et al., 1983). Then, this ranking was used to find the group requirements of the cows based on the average and standard deviations of the NE_L and MP concentration of the cows in the group.

Cows' requirements were directly used to estimate the NE_L and MP concentration of the group diet. First, the average and SD of NE_L (and MP) concentration of the group were calculated. Then, a factor of SD was used to add to the average obtained concentration of NE_L and MP. Tested factors were 0xSD, 0.5xSD, or 1xSD.

This grouping strategy was applied to the herd every month to regroup all the lactating cows (optional groups) based on their current requirements. Allocation of cows to optional groups was also designed to maximize the IOFC (Cabrera et al., 2012) in each regrouping event. This process involved iterations over all permutations of optional groups and cows in the group to calculate the potential IOFC of cows considering the current group arrangement. An effect of milk loss, for a predetermined number of days, for cows moving to a new group was an optional consideration within the simulation framework.

6.5.4. TMR

This was the object in the model that contained the information regarding the TMR fed to each group at each regrouping during the simulation time, which was set at the regrouping time (every month). More specifically, TMR was a list of NE_L and MP provided in the diet for each nutritional group every month.

6.6. Model Verification and Validation

After model development, the model behavior was checked. Verification and validation of the model components were performed in an iterative manner. The model was rechecked multiple times to gain confidence that it can be used to study the system. Through verification the model was proven to do, correctly, what it is expected to do (Sorensen, 1990). Verification can be understood simply as a debugging process in computer jargon. Here, a unit test method was used to verify the correctness and functionality of each component of the model.

Validation of a model checks the degree of agreement between the model and the target system (Sorensen, 1990). Usually, validation is considered to be the more difficult and therefore, both objective and subjective methods, are to be introduced and used to test the system behavior (Sorensen, 1990). Objective validation of the model would include statistical tests to find the degree of agreement between the model outputs and the real farm performance (e.g., goodness of fit tests). However, in practice, it might be unfeasible to perform a field trial in parallel to the model. Thus, as it happens in most livestock models in the literature (Sorensen, 1990), a subjective validation of the model was used. For this purpose the model was run for a year and the results of 1,000 replications of 5 herds were summarized. These results were contrasted against original data, industry averages, and authors' expert opinions. Visual graphs were used to help in this process.

6.7. RESULTS AND DISCUSION

Functionality of scheduling involuntary culling and death were tested against the input empirical distributions. The density plot of culling due to death and mastitis created from 1,000 iterations of a 1,460-cow herd is depicted in Figure 6.3. It is obvious that the density plot created from the model is similar to the input distribution shown in Figure 6.1 Panel A. Similar tests were applied to all the important components of the model to verify that the model does what it was expected to do.

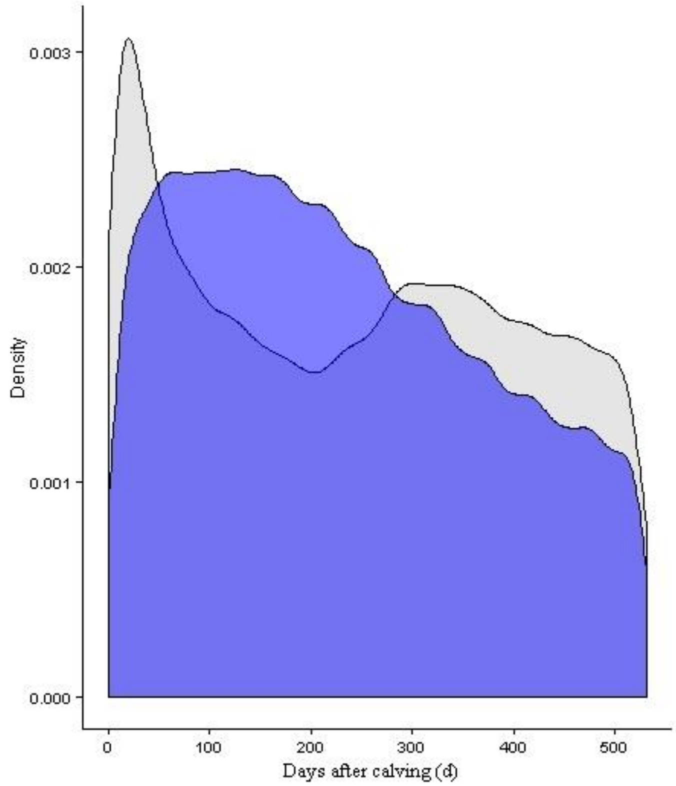

Figure 6.3. Density plot of culling due to death (light shade) and mastitis (dark shade) resulted from 1,000 replications of a 1,460-cow herd.

For validation purposes, summary statistics of the herds after a year is provided in Table 6.3. Herd summary statistics of the model projections after a year of daily iterations are comparable with the statistics of the input herds and industry averages. Note that the average herd milk production in Table 6.2 (input values) and Table 6.3 (model results) are considerably different from each other. This was done, by using the same milk production function parameters for all the herds, to limit the effect of milk production on the economic results of the model. The variations in the rolling herd averages among the herds as observed in Table 6.3 are due to the individual cow milk production potential differences among the herds. Other parameters in Table 6.3 are within plausible ranges in the industry and the model appears to produce reasonable outcomes. The most important unavailable data that could have a great impact on the projections include: 1) individual farm culling pattern and frequencies, 2) culling policies related to reproductive failure, and 3) specific reproductive program used on the farm. Even with these limitations, reproductive parameters such as average days in pregnancy and average estrous detection and 21-d pregnancy rate were close to the input herd values (Table 6.2). In general, the herd management and practices (like herd expansion) are not known to the model and the similar statistics of the herd might not be expected after a year of simulation.

Table 6.3. Herd structure and dynamics (mean (SD)) after a year over 1,000 replications of different input herds

	Herd size (lactating + dry)				
Parameters	331	570	727	787	1,460
Rolling Herd Average (kg/cow per yr)	11,980	12,149	12,072	12,644	12,309
Average DIM (d)	166 (4.73)	164 (3.88)	167 (3.66)	158 (3.29)	170 (2.61)
Average DIP (d)	130 (5.94)	134 (4.49)	133 (3.58)	132 (4.06)	141 (2.47)
Average lactation (#)	2.18 (0.06)	2.02 (0.04)	2.34 (0.05)	2.22 (0.04)	2.01 (0.03)
1st lactation (%)	38 (2)	43 (2)	37 (2)	40 (2)	43 (1)
2nd lactation (%)	28 (2)	28 (1)	27 (1)	26 (1)	29 (1)
3rd lactation (%)	20 (2)	17 (1)	17 (1)	17 (1)	16 (1)
>3rd lactation (%)	14 (1)	12 (1)	19 (1)	17 (1)	12 (1)
Average estrous detection (%)	50 (4)	58 (5)	50 (5)	54 (6)	45 (9)
Average 21-d PR (%)	18 (1)	18 (2)	19 (2)	19 (2)	18 (3)

Another part of the model, which was highly related to the effects of the nutritional grouping, were the modules calculating BW and BCS of the lactating cows. Both of these were affected by BE, which depended on the energy consumed and the stage of the lactation. The average changes (over 1,000 replications) in BW and BCS of the cows fed only 1 nutritional group are illustrated in Figure 6.4, Panel A and B. The shape, the trend, and the values for BW and BCS are similar to the ones depicted in NRC (2001) and the curves obtained from using the Korver function for BW prediction (Cabrera et al., 2012).

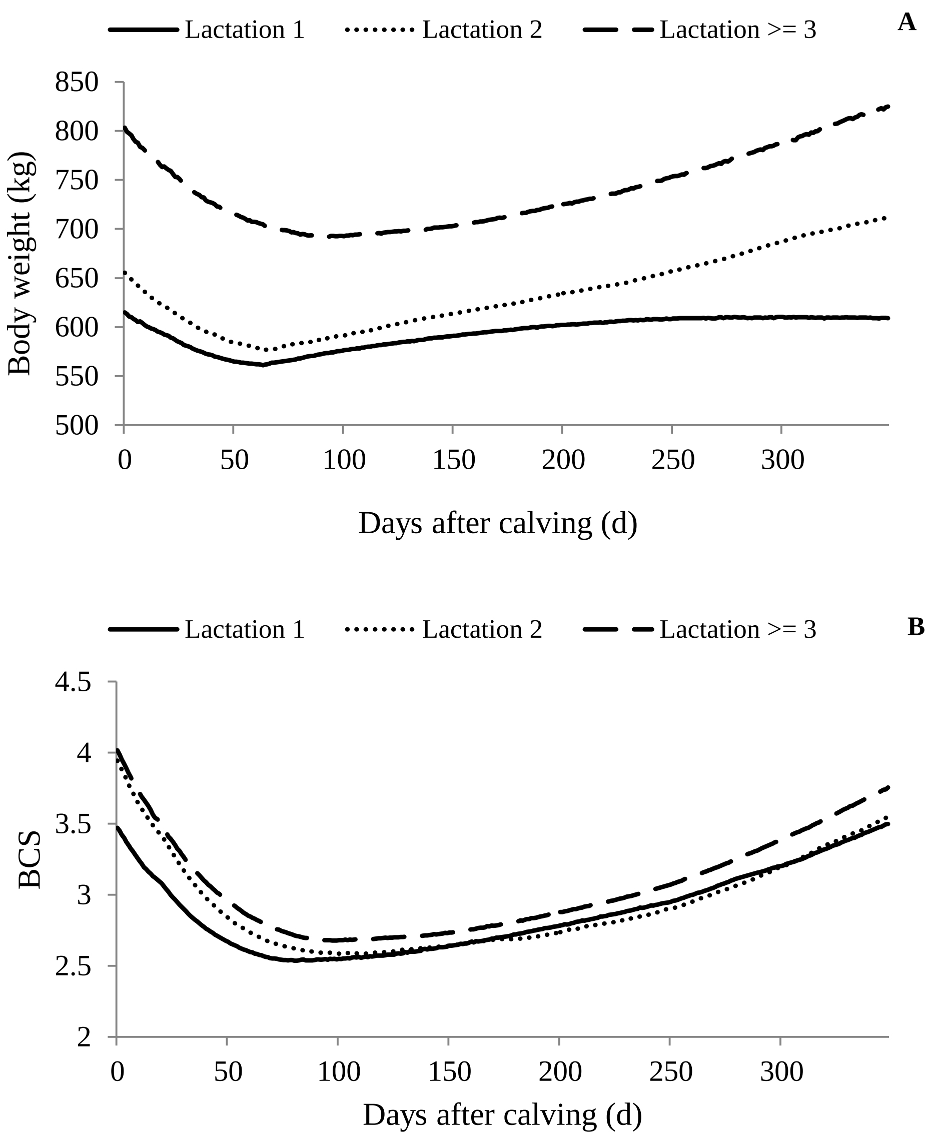

Figure 6.4. Panel A: Average body weight change after calving for first, second and ≥ 3 lactation cows. Panel B: Average body condition score (BCS) change for first, second and ≥ 3 lactation cows obtained from 1,000 replication of a 1,460-cow herd with 1 nutritional group.

As previously mentioned NE_L and MP concentration of the group's diet was obtained based on the group's average plus a SD factor (0, 0.5, and 1). Increased NE_L concentration of the diet above the average (i.e., 0.5xSD and 1xSD) resulted in shifted and skewed distribution of BCS as illustrated in Figure 6.5. This figure represents density plots of the BCS of 1,000 iterations of the cows in the 331-cow herd at the end of simulation for 3 nutritional groups. It is clear that the cows fed on a diet with greater than the average NE_L concentration increased their body energy content, which is reflected in their BCS. This is due to the way the model handles the energy, which is accounting for energy intake in the form of body energy without any changes on the milk production or DMI of the cow. In this model to control the BCS range an arbitrary bound of 2.0 and 4.5 were selected, and the milk production and DMI of the cows that would go beyond the limits were affected appropriately (explained in the materials and methods). Based on density plots of BCS in Figure 6.5 the average NE_L (0xSD) which produced a normal shaped BCS distribution with the mode and average BCS around 3.25 was chosen for the rest of this and the following companion study (Chapter 6 Section 2). The MP concentration of the diet does not affect the body energy of the cows and therefore different factors of SD could be used. The effects of using different factors of SD for MP are important for nutritional grouping strategies and therefore included as sensitivity analyses in the companion study (Chapter 6 Section 2). In the remaining of this study 1xSD above average for MP was used to validate the results.

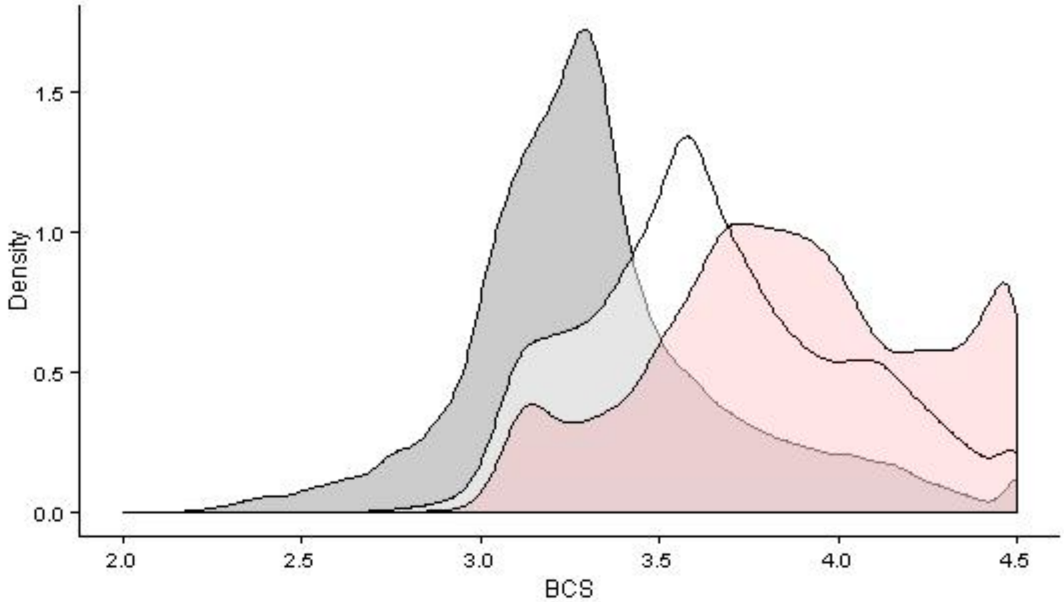

Figure 6.5. Body condition score (BCS) density plots under 3 different NE_L concentration of the diet for 3 nutritional groups of a 331-cow herd at the end of a 1-yr simulation with 1,000 iterations. Average NE_L concentration of the group + 0xSD (grey shade; leftmost distribution), + 0.5xSD (white shade; middle distribution), and + 1xSD (rightmost distribution). Total area under the curves adds up to 1.

Average NE_L concentration of the offered diet from 1,000 iterations of the 331-cow herd over the simulation time and for 1 and 3 nutritional groups is demonstrated in Figure 6.6. The NE_L of the offered diet was set as the average NE_L concentration of the requirements of the group of the cows and the average MP + 1xSD of the group. Having 3 nutritional groups clearly separated the offered diet into 3 batches: high, mid, and low groups, which are closely targeted to the cows in the group to meet their requirements. Although, the 3 nutritional groups offered 3 distinct NE_L concentration of the diet for each group, the average NE_L concentration offered across 3 nutritional groups is similar to the 1 nutritional group diet. This is an expected model behavior that was clearly portrayed throughout time. The fluctuations across simulation time are due to changes in the herd structure, specially the changing proportions of fresh and dry cows in the herd.

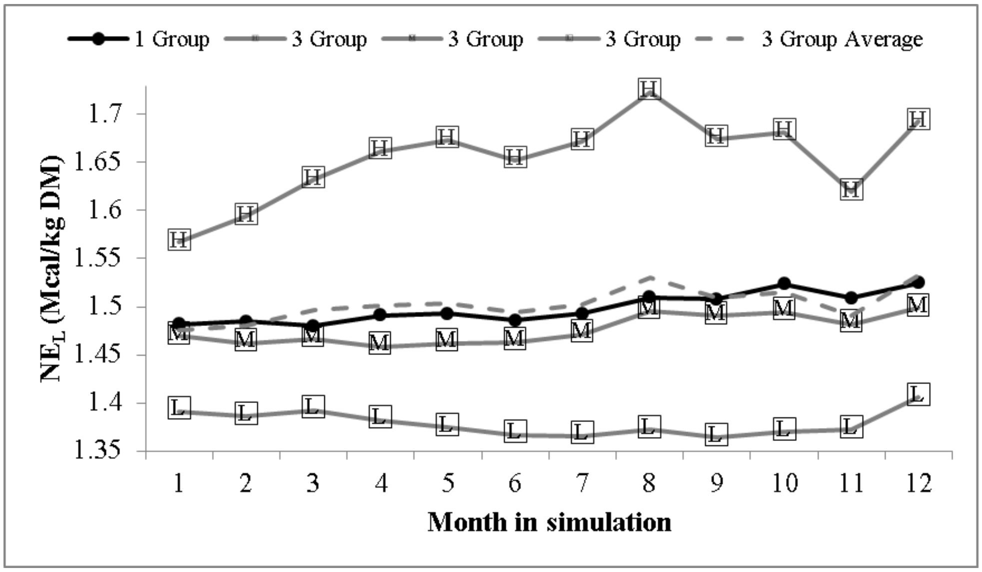

Figure 6.6. Simulated average NE$_L$ concentration of the offered diet between 1 nutritional group and 3 nutritional groups (marked as H=high, M=mid, and L=low groups) obtained from 1,000 iterations of a 331-cow herd regrouping cows monthly. The diet NE$_L$ concentration was set as the average NE$_L$ concentration requirements of the group. Dashed line is the average NE$_L$ concentration of the diet of the 3 nutritional groups. Marker points indicate the monthly regrouping occasions when diet was reevaluated.

The changes in average concentrations of NE$_L$ and MP of the diet across 1,000 iterations throughout 350 d postpartum with 3 nutritional groups are illustrated in Figure 6.7. As expected, NE$_L$ and MP concentration offered in the diet decreased gradually across days postpartum due to the lower cow requirements later in lactation. These figures confirm that the diet concentration and grouping modules work properly.

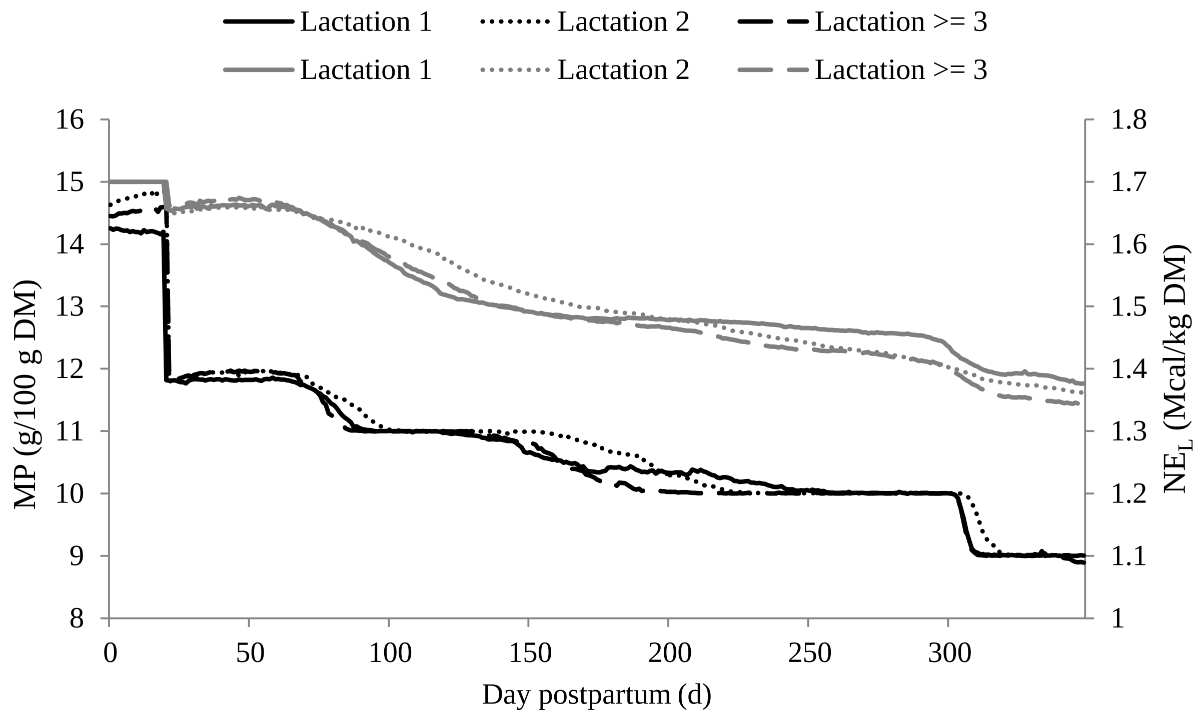

Figure 6.7. Average NE_L (light shade) and MP (dark shade) concentrations of 3 diets from 1,000 iterations provided to cows throughout days postpartum in a 331-cow herd

6.8. CONCLUSIONS

A stochastic Monte Carlo simulation was developed to study the impact of nutritional grouping in dairy cows. The type of the simulation and the methodology of cow-individual daily energy balance are unique features of the current model compared to previous simulation studies in nutritional grouping. The developed model was verified for correctness and it was subjectively validated. Based on these, the model seems to give consistent and plausible results and can be used to evaluate the economic impacts of nutritional grouping in dairy cows.

Section 2

Economic impact of nutritional grouping in dairy herds

A. S. Kalantari, L. E. Armentano, R. D. Shaver and V. E. Cabrera

6.9. ABSTRACT

This manuscript evaluates the estimated economic impact of nutritional grouping in commercial dairy herds using a stochastic Monte Carlo simulation model developed and described in a companion paper. The model was initialized by separate datasets obtained from 5 commercial dairy herds. These herds were selected to explore the effect of herd size, structure, and characteristics on the economics and efficiency of nutrient usage according to nutritional grouping strategies. Simulated status of each cow was updated on a daily basis together with the nutrient requirements of net energy for lactation (NE_L) and metabolizable protein (MP). The amount of energy consumed directly affected the BW and BCS changes in the model. Moreover, to control the range of observed BCS in the model constraints on lower and upper bounds of BCS were set. Cows that would drop to BCS < 2.0 were assumed to stay at BCS = 2.0, but at a decreased milk production (New daily milk (kg/d) = daily milk (kg/d) – (deficient NE_L intake (Mcal/d)/NE_L in milk (Mcal/kg))), and cows that would reach BCS > 4.5 were assumed to stay at BCS = 4.5, but at decreased DMI (New DMI (kg/d) = DMI (kg/d) – (extra NE_L intake (Mcal/d)/dietary NE_L (Mcal/kg))). Each month, lactating cows were regrouped to more homogeneous groups of cows according to their nutrient concentration requirements. The average NE_L concentration of the group and a level of MP (average MP, average MP+0.5xSD, or average MP+1xSD) were considered to formulate the group diet. The calculated income over feed costs (IOFC, $/cow per yr) of having more than 1 nutritional group among the herds ranged

from \$33 to \$58, with an average of \$39 for 2 groups and \$46 for 3 groups, when group was fed at average NE_L concentration and MP concentration = average MP+1xSD. The improved IOFC was explained by increased milk sales and lower feed costs. Higher milk sales were a result of fewer cows having a milk loss associated with low BCS in multi-group scenarios. Lower feed costs in multi-group scenarios were mainly due to less RUP consumption. The percentage of total NE_L consumed captured in milk for greater than 1 nutritional group was slightly lower than 1 nutritional group due to better distribution of energy throughout the lactation and higher energy retained in body tissue, which resulted in better herd BCS distribution. The percentage of N fed captured in milk increased with more than 1 group, and was the most important factor for improved economic efficiency of the grouping strategies.

Key words: stochastic, simulation, nutritional feeding, economics

6.10. INTRODUCTION

Grouping cows is a common practice farmers use to manage their herds more efficiently. Farmers may use various grouping strategies to separate dry cows with remote or proximal expected calving dates, cows that have calved recently, sick cows, pregnant cows and first or later lactation cows. Grouping addresses cow-specific needs (Cabrera and Kalantari, 2014). However, grouping lactating cows for nutritional purposes and providing them with more precisely formulated diets is not an extensively adopted strategy (Contreras-Govea et al., 2015), despite the fact that many studies have shown its possible economic advantage (Coppock et al., 1974; McGilliard et al., 1983; Williams and Oltenacu, 1992; Pecsok et al., 1992; Stallings, 2011; Cabrera et al., 2012). Reasons that farmers do not favor nutritional grouping can be attributed to facility management limitations (such as machinery and facilities), labor cost, difficulty in managing multiple diets, and the presumption of milk production loss associated with group changes (Contreras-Govea et al., 2013, 2015). Even when farms may have the facilities and may be grouping cows, which would have different nutritional needs, all cows are still fed with a common TMR (Contreras-Govea et al., 2015). Hutjens (2013) suggests that farmers may also be concerned about multiple nutritional diets introducing errors in the formulation and delivery of rations.

Total mixed rations have become an industry standard for feeding management and many dairy farms are using just 1 TMR for all the lactating cows despite major differences in nutritional requirements of dairy cows in different lactation stages (Allen, 2008). For example, 58% of Wisconsin and Michigan dairy survey farms used the same TMR for all lactating cows (Contreras-Govea et al., 2015). The adoption and application of a single TMR as an industry standard has resulted in more over-conditioned cows in some cases, and greater nutrient excretion issues (Allen, 2009). Cows at similar lactation stages could also have different nutritional requirements because of their productivity due to their genetic potential. When feeding only 1 TMR diet, it is usually formulated for high-producing cows to ensure these cows reach their full milk production potential, which results in over-feeding lower producing cows. Therefore, 1 TMR diet can result in more over-conditioned cows, higher nutrient excretion in the manure, and increased costs of nutrient usage. A strategy to relieve this problem is adopting nutritional groups with more precise diets, which could improve herd health, decrease environmental concerns, and increase income over feed costs due to the better tailored diet to the cow requirements in a group. More precise diets would also improve milk productivity (Bach, 2014). Therefore, there is a need to systematically quantify the possible economic value and nutrient use efficiency of grouping lactating cows under variable herd conditions. Consequently, the objective of this study was to evaluate the economic effects and the nutrient use efficiency of nutritional grouping in 5 commercial herds.

Previous studies (Stallings and McGilliard, 1984; Stallings, 2011) have used lead factors or the levels at which diets should be formulated for a group of cows to determine the energy and protein concentration of the diet for the group. This method uses group average milk production and predicted DMI to determine the group requirements. In this study, having the estimated nutrient requirements of every cow in every group, the NE_L and MP concentrations were directly used to formulate the diet based on the group average and standard deviation of their requirements (Schucker et al., 1988).

Thus, objective of this study was to evaluate the economic value of nutritional grouping on inputs from 5 commercial dairy herds using dynamic, stochastic simulation model developed and explained in the previous section (Chapter 6 Section 1). Furthermore, the use of nutrient concentration requirements was demonstrated to formulate more precise diets within defined nutritional groups.

6.11. MATERIALS AND METHODS

6.11.1. Simulation Framework

A dynamic stochastic Monte Carlo simulation model was developed to simulate each individual cow in a herd to study different nutritional grouping strategies and is described in detail in the previous section (Chapter 6 Section 1).

After scheduling all the events, every cow in the herd was evaluated on a daily basis to check if the cow matched to any specific event on a given day postpartum. If a match was found, the cow information was updated. For example, if a cow was evaluated at 90 d postpartum and the cow was scheduled to be pregnant on that day, then the cow information was updated to be pregnant. In the cases when the event involved culling or mortality, replacement with a heifer at the time of first calving was assumed to occur immediately. Immediate replacement maintained the herd size constant. The detailed process of the modeling and the used distribution parameters are described in the previous section (Chapter 6 Section 1).

6.11.2. Cow Attributes

Cow attributes in the model included: 1) daily milk, fat and protein production, 2) BW and BCS, 3) and DMI. Wood's function (Wood, 1967a) was used to estimate daily milk productions, and fat and protein percentages. Dry matter intake was calculated on an individual basis and based on the NRC (2001; p. 4) DMI equation. Changes of BW and BCS were calculated based on the estimation of body energy, which was directly affected by the amount of energy consumed by the cow. Constraints on lower and upper bounds of BCS were used to control the range of observed BCS in the model. A BCS = 2.0 was set as the lower bound: cows that would drop to BCS < 2.0 were assumed to stay at this BCS, but at decreased milk production (New daily milk (kg/d) = daily milk (kg/d) – (deficient NE_L (Mcal/d) / NE_L in Milk (Mcal/kg))). A BCS = 4.5 was set as the upper bound: cows that would reach BCS > 4.5 were assumed to stay at this BCS, but at decreased DMI (New DMI (kg/d) = DMI (kg/d) – (extra NE_L (kg/d) / dietary NE_L (Mcal/kg))). More details of the methods for obtaining each of these attributes can be found in Chapter 6 Section 1.

6.11.3. Cow Nutrient Requirements

Appropriate equations from NRC (2001) were used to calculate the net energy and protein requirements of dairy cows. Net energy requirements of maintenance, milk production, and growth were aggregated to find the total NE_L requirements. Metabolizable protein is more representative of the protein value of the diet and the cow's specific requirements (Varga, 2010) and was used instead of CP. Estimation of MP requirements was also based on maintenance, milk production, and growth (Chapter 6 Section 1).

6.11.4. Grouping Dynamics

Obligated and optional groups were defined to group the cows in the herds. A group of cows shared a common diet. Obligated groups consisted of 2 groups, one for dry cows and another for fresh cows (under 22 d postpartum). These were the groups that were fixed in all the studied herds. Cows in the fresh group were fed a constant NE_L of 1.7 Mcal/kg and the NE_L for dry group was 1.28 Mcal/kg. The MP concentration requirements (g/100 g DM) for fresh cows were set based on the 83rd percentile of the cows in the group to ensure adequate protein consumption. The MP concentration of the dry cow group was fixed at 7 g/100g DM. On the other hand, optional groups were used to explore the effect of nutritional grouping on the overall milk income over feed costs (**IOFC**), nitrogen excretion and efficiency. The feed cost calculation included RDP, RUP, and energy costs per kg of DM. Cows were considered for a group change at the end of each month for optional groups and whenever needed for obligated groups (further details in Chapter 6 Section 1).

The number of the optional groups was variable and was altered to test the economic effects of a number of nutritional groups in different herds. To be consistent among herds, the sizes of optional groups were chosen to be approximately equal among them (total available cows were divided by the number of defined nutritional groups). Optional group size was set at model initialization. The model first assigned a maximum size based on the total number of available cows in the herd to all the optional groups. Later, the model followed the cows and moved them among the groups as required. Consequently, throughout the simulation, the number of cows in each group could vary from the initial starting point because of cow movements from optional groups to obligated ones or vice versa. Thus, it was assumed the group size changes continually based on the status of available cows in the herd.

The monthly regrouping process to optional groups started by ranking the cows based on their NE_L and MP requirements (clustering method; Chapter 6 Section 1). Then all permutations according to current group sizes were determined. The model iterated over all the permutations and calculated the initial IOFC for each arrangement of the cows inside the group. This last process was used to assign the cows to groups in a way that maximized the IOFC of the herd at that point in time. Inside this iteration the average NE_L and MP concentration of the requirements were calculated for the group. Then, the diet for the group was formulated for average NE_L requirements of the group and the maximum NE_L concentrations for these groups were set at 1.79 Mcal/kg. Different levels of NE_L concentrations, average NE_L, average NE_L+0.5xSD and average NE_L+1xSD, were considered in Chapter 6 Section 1. It was found that formulating the diet for above the average NE_L concentration changed the body energy contents of the cows in the herd, resulting in an undesirable proportion of obese cows in the herd. For that reason, in this study, only average NE_L concentration was used. Regarding MP, the base scenario used MP+1xSD, but since MP concentration does not affect BCS, 2 other scenarios were also studied; average MP and average MP+0.5xSD.

6.11.5. Case Study Herds and Projection Timeline

The 5 sets of initial herd characteristics for this case study are summarized in Table 6.4. Holstein herds from Wisconsin with different sizes were chosen. The size of the herds was an important criterion in this study, which ranged from 331 to 1,460 adult cows. All herds used a TMR feeding management system. Variables included in the herd input files were: cow id, parity, days postpartum, days in pregnancy, fat percentage, protein percentage, 305 mature equivalents (**ME 305**), BW (if available), BCS (if available), and predicted producing ability (**PPA**) -producing ability of cow in future lactations- (if available). Using these variables the model captured current herd profiles (d = 0 of the simulation) and then projected individual cow and herd performance daily for a year (d = 365).

Table 6.4. Input dairy herd characteristics and structure at the beginning of the simulation (d=0).

	Herd Size (Lactating + Dry)				
Characteristics	331	570	727	787	1,460
Average Herd ME305[1] (kg/cow per yr)	13,348	16,140	13,897	12,884	14,188
1st Lactation (%)	38	43	39	39	45
Average days in milk[2] (d)	193	169	181	165	174
Average days in Pregnancy (d)	134	140	141	133	157
Average lactation number (#)	2.03	1.99	2.29	2.21	2.02
21-d Pregnancy Rate[3] (%)	17	18	19	19	18
Conception Rate[3] (%)	35	32	36	37	40
Estrus Detection[3] (%)	49	57	51	51	45
Culling[3] (%/yr)	35	32	36	37	40
Abortion[3] (%/gestation)	16	7	11	11	7
Cow BW available	Yes	No	Yes	No	No
Cow BCS available	No	No	No	No	No

[1] 305 d mature equivalent milk production

[2] Average days in lactation

[3] As defined and calculated in DairyComp305 (Valley Agricultural Software, Tulare, CA)

6.11.6. Economic Parameters

Economic parameters for the base scenario of calculating IOFC were set as 10 years Wisconsin average prices from 2005 to 2014. Thus, milk price was set to $0.39/kg of milk. For feeds prices, corn and soybean meal with available 10 years historical prices were used to set all other feed commodity prices. A current (April 2015) relative price of 24 commodities in relationship to corn and soybean meal was calculated and used to set their 10 years historical average price. Ten-year historical average (2005-2014) of 26 default commodities in the FeedVal 6.0 model (http://DairyMGT.info: Tools) was used to calculate the nutrient prices of NE_L, RDP, and RUP. The calculated nutrient prices were: \$0.1/Mcal of NE_L, \$0.18/kg RDP, and \$1.04/kg RUP. These values were used as the typical price for the base scenario runs.

6.11.7. Scenario Analyses

To explore the impact of different milk and nutrient prices on the economic value of nutritional grouping, extreme differences between these input variables were explored. For this purpose two extreme scenarios were developed. The worst case scenario was designed by coupling the lowest milk price with the highest nutrient costs and vice versa for the best case scenario. Ten-year annual average of milk price was used to set the highest (\$0.52/kg) and lowest milk (\$0.29/kg) price. For the nutrient costs the same approach, as described above, was used to find the highest and lowest nutrient prices. The highest (lowest) nutrient costs were set at \$0.14/Mcal of NE_L (\$0.05), \$0.26/kg RDP (\$0.09), and \$1.52/kg RUP (\$0.52).

Furthermore, possible milk loss due to regrouping lactating cows was also explored by assuming milk losses of 1.82 kg/d for 5 days (Cabrera and Kalantari, 2014). In addition, the effect of having first lactation cows as a separate nutritional group was explored as another scenario–as this is a common practice in a number of commercial herds (Contreras-Govea et al., 2015). In this scenario, the first lactation cows was an obligatory group and were fed based on the nutrient concentration requirements of the cows in the group similar to the optional groups.

6.12. RESULTS AND DISCUSSION

6.12.1. Grouping

Lactating cows of the optional groups after the fresh period (> 21 d postpartum, 592 cows) from a 787-cow herd at 300 d in the simulation are shown in Figure 6.8 ranked according to their NE_L concentration requirements. It is clear that lactating cow requirements vary substantially on a given day because of differences in d postpartum, pregnancy status, BW, and milk production. In this example, the highest NE_L concentration requirement was from a cow in 3rd lactation, 23 d postpartum, with a milk yield 20% above herd average. The lowest NE_L concentration requirement was from a cow in 3rd lactation, 385 d postpartum with 10% below average milk yield. To cope with this high variability, precision feeding, according to an individual cow requirements would be ideal, but unfortunately is not yet practical, especially in larger herds (Sniffen et al., 1993). On the other hand, preparing a diet of just 1 TMR for all of the cows could result in large overfeeding or underfeeding problems. A diet is usually formulated for the high-producing cows to ensure that the milk production is maintained (Weiss, 2014), but feeding the group with just 1 TMR is inefficient from a nutrient use stand-point. A practical way to overcome this high variability is to group them according to their requirements. The effect of grouping these 592 lactating cows (from 787-cow herd) is illustrated in Figure 6.9 where the difference between offered and the required NE_L concentrations (offered NE_L – required NE_L, Mcal/kg) are depicted for 3 cases of nutritional groupings. Figure 6.9, Panel A shows that when feeding all the cows as one group and formulating the diet based on the average NE_L concentration of the group approximately half of the cows are being overfed and the other half underfed. However, it should be noted that the average NE_L concentration of the requirements are not necessarily normally distributed. Thus, the average NE_L concentration does not always result in over-feeding half the cows and under-feeding the other half. It was observed that the distribution was heavily affected by the herd structure at the point of regrouping the cows. Specifically, it depended on the percentages of fresh animals that were moving into optional groups (> 21 d postpartum) -cows with the highest requirements-, which caused right skewedness in the distribution. Also it was dependent on the percentages of late lactation cows moving to the dry group, which caused left skewedness in the distribution. Figure 6.9, Panel A also shows that increasing the number of groups decreases the variability among the cows within the group, which is especially beneficial in the case of large herds, and when the distribution of the requirements are not normal (McGilliard et al., 1983).

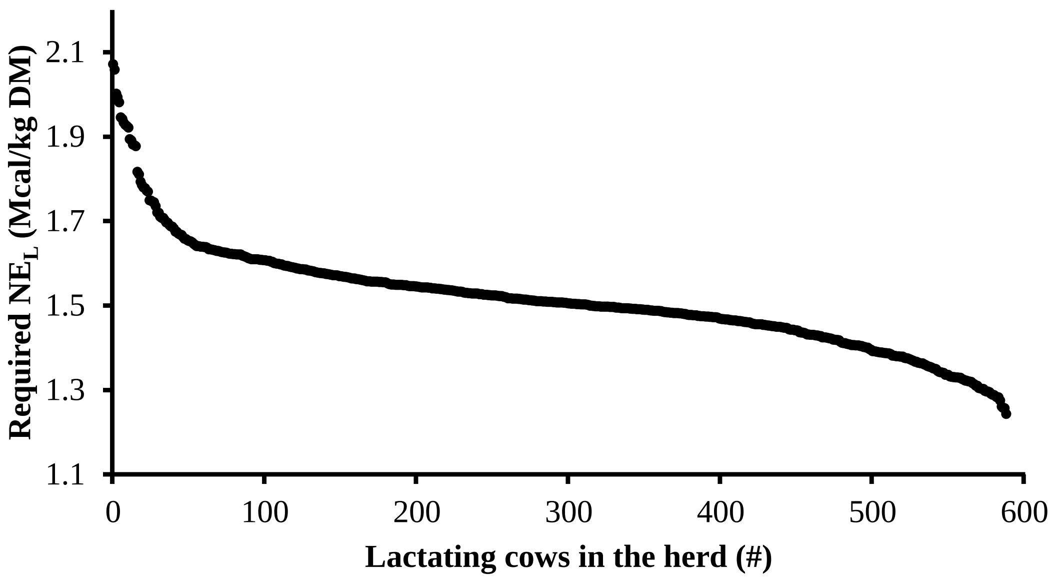

Figure 6.8. NE_L concentration of the requirements of 592 lactating cows (all lactating cows minus fresh cows < 22 d postpartum) from the 787-cow herd at d=300 of simulation.

Figure 6.9, Panel B shows the difference between offered MP and the required MP for the cows in the group when feeding the group of cows average MP+1SD. In this case, because cows receive 1xSD above the average MP concentration, the number of cows underfed MP is much smaller while the number of cows overfed is much higher than in the case of NE_L. All additional discussion points for the NE_L also apply to the MP.

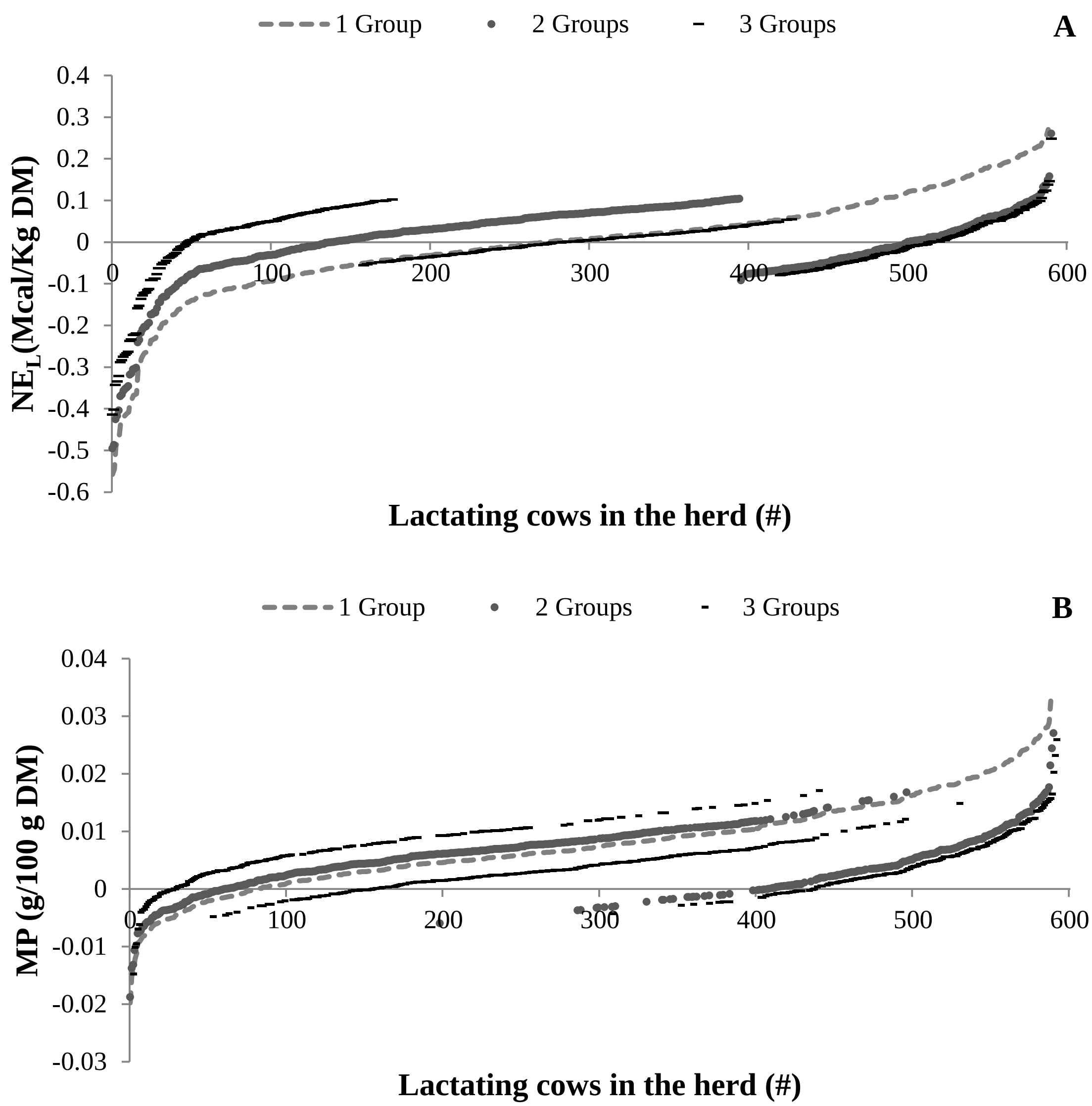

Figure 6.9. Panel A: Difference between provided and required NE_L concentration under 1, 2, and 3 nutritional groups based on the diet offered at the average NE_L concentration of the group. Panel B: Difference between provided and required MP concentration under 1, 2, and 3 nutritional groups based on the diet offered at the average MP+1xSD of the group. Both charts are for 592 lactating cows (all lactating cows minus fresh cows < 22 d postpartum) from the 787-cow herd at d=300 in simulation.

The economic value of nutritional grouping measured in terms of IOFC is displayed as the difference from 2 to 4 nutritional groups and 1 nutritional group for different levels of MP provided in the diet (average MP, average MP+0.5xSD, and average MP+1xSD) in Figure 6.10. It is clear that there is an economic gain from nutritional grouping. These gains depended on the number of groups and varied from ($/cow per yr) $39 for 2 groups to $46 for 3 groups, to $47 for 4 groups when the diet had average MP+1SD concentration (Figure 6.10). This gain also depended on the MP concentration in the diet, and it was the lowest when the cows were offered an average MP concentration of the group (approximately $15/cow per yr) and highest at average concentration MP+1xSD (approximately $44/cow per yr). This was mainly due to the fact that the RUP had the highest nutrient cost and offering a greater concentration of it in the diet would accentuate even more the potential benefit of applying nutritional groups. For the rest of the manuscript the 1xSD above average of MP concentration of the requirements is used to illustrate the effect of grouping cows.

The gain in IOFC with more nutritional groups was due to the higher milk production and lower feed costs. Higher milk production for more than 1 group was due to less cows having milk loss for low BCS (BCS < 2.0). The lower feed costs in 2 and 3 groups were mainly due to less RUP cost (Figure 6.10). Compared to RUP cost, other components of IOFC (RDP and NE_L costs and milk revenue) were more stable across different grouping numbers and MP concentrations in the diet.

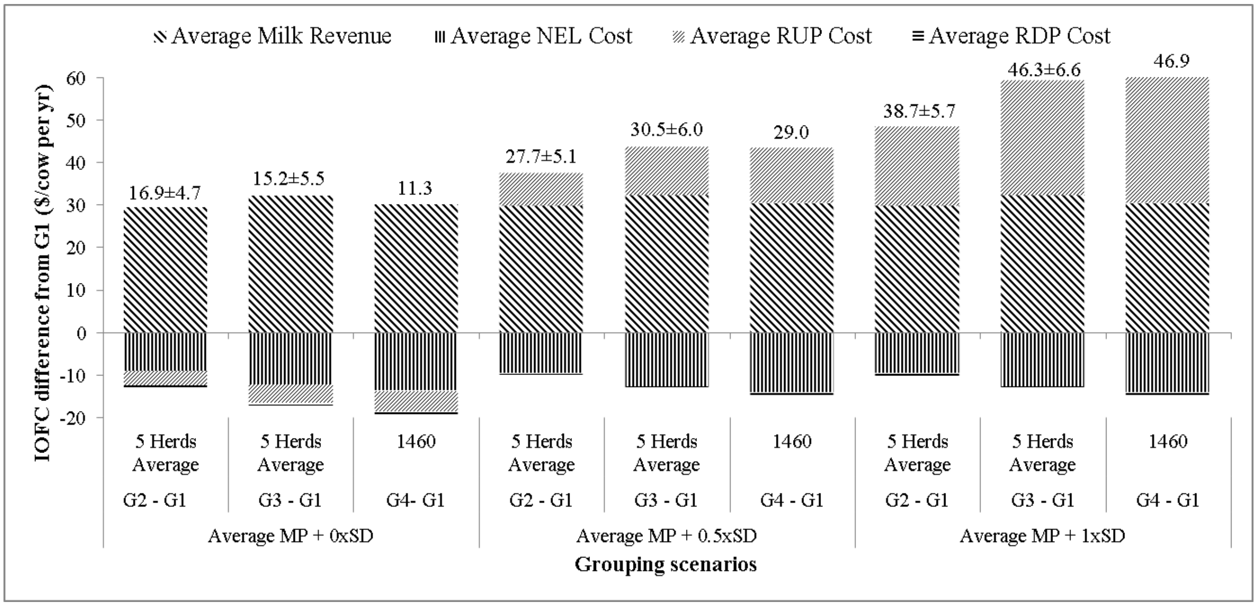

Figure 6.10. Average difference in income over feed cost (IOFC) of 2, 3, and 4 groups (G2, G3, and G4) and 1 group (G1). The average difference in IOFC is disaggregated in its components, which are rumen degradable protein (RDP) cost, rumen undegradable protein (RUP) cost, NE_L cost, and the milk revenue. The zero line is the average IOFC obtained by 1 group under different MP scenarios and were equal to $2,883, $2,852, and $2,822 for diet formulated at average MP, average MP+0.5xSD, and average MP+1xSD, respectively. The labels on top of the bars are the average extra IOFC ± SD (SD among the herds) above 1 group. 4 nutritional groups were just applied to the largest herd (1,460-cow herd).

Regardless of herd size, the largest relative IOFC gain was obtained when moving from 1 group to 2 groups. After 2 groups the gain was lower. Comparing 1 group and 2 groups, the IOFC gain ranged ($/cow per yr) from $33 (570-cow herd) to $49 (787-cow herd) when average MP+1xSD. The overall (average of 5 herds in the study) gain in IOFC ($/cow per yr) from 1 group to 2 groups was $39±6 and from 1 group to 3 groups was $46±7 (Figure 6.10). Economic gains found in other studies are different because of differences in the model and the input values used in the other studies. For example, Williams and Oltenacu (1992) reported that the mean annual IOFC ($/cow per yr) of 3 nutritional groups were $21, $33 and $40 higher than 2 groups at production levels of 8,000, 9,000 and 10,000, respectively. In this study, the average gain over all 5 herds from 2 groups to 3 groups was ($8/cow per yr), which is lower than the reported values in Williams and Oltenacu (1992). This might be due to the fact that Williams and Oltenacu (1992) included the effect of different milk production levels in the herd, but here, to focus on the value of grouping, we tried to exclude the effect of different milk production as much as possible. Here, most of the economic gain was observed in the first grouping (1 group to 2 groups). St-Pierre and Thraen (1999), using an economic optimized lead factors for CP and NE_L for different group numbers, calculated an average economic gain ($/cow per yr) of $44 and $77 when comparing 2 and 3 groups with 1 group, respectively. These values are comparable to the ones found in this study in the case of offering a diet of average MP+0.5xSD and average MP+1xSD. A study by Østergaard et al. (1996) used a dynamic stochastic simulation model to compare different grouping strategies under different reproductive and culling management. In that study, the Scandinavian Feed Units (SFU) was used. Regrouping of the cows was triggered by drop in milk production levels after 24 weeks post calving. Thus, using SFU and considering the setting of that study, feeding of the cow was not according to the calculated nutrient requirements, but was specified by a feeding regime of TMR with up to three different groups (Østergaard et al., 1996). Although, the differences in the feeding systems make it difficult to compare the current study with the study by Østergaard et al. (1996), they also showed that overall 1 group was inferior to other grouping strategies mainly due to the economic effect of lower milk production and higher amount of concentrate intake in 1 group (Østergaard et al., 1996). They also found that marginal net revenue per cow per year was lower under 1 group compared to 2 or 3 groups under all scenarios of milk production and reproductive and culling management. However, the economic difference due to good and bad reproductive and culling management was larger (Østergaard et al., 1996).

It should be noted that, in this study, we used the actual requirements of the cows to determine the offered diet concentration of NE_L and MP, and therefore no lead factor was used. This had a great impact on the overall economic value of a grouping strategy. We also speculate that not considering the herd structure and the dynamics of the herd throughout lactation, as was done in this study, could overestimate the economic gain of nutritional grouping. The average economic gain for 3 groups compared with 1 group without herd dynamics was reported to be $396/cow per yr in Cabrera et al. (2012).

The other important factor in economic evaluation of grouping lactating cows is the extra labor needed to formulate, prepare, deliver, and the extra costs of running the mixers for preparing the TMR for each group separately. In addition, there is labor cost related to moving the cows among groups. These costs are specific to the herds and vary dramatically among the herds and were not included in the current study. However, with current feed and milk prices it is reasonable to expect that even after including extra costs for labor and management of grouping, having more than 1 group could be beneficial. Overall, profitability and feasibility of nutritional grouping is highly farm and market dependent. The farm size has an impact on the feasibility of nutritional grouping. For example, in larger herds the extra labor for regrouping and moving cows might be less important than in smaller herds (Østergaard et al., 1996). Also, when market conditions determine high feed costs and low milk price, nutritional grouping could be more economically appealing (Allen, 2008; Hutjens, 2013). Simulation studies (Williams and Oltenacu, 1992; Pecsok et al., 1992) have suggested dividing up the lactating cows into 3 nutritional groups for optimal efficiency. Results from this study corroborate those previous reports indicating that economic gain and efficiency were increasing until 3 nutritional groups.

Also, it was observed that regardless of the herd size, having 2 nutritional groups for lactating cows provides the highest relative economic gain.

6.12.3. Formulated Diet

The average $NE_{L,}$ RDP, and RUP concentrations in the DM under 3 levels of offered MP concentrations are summarized in Table 6.5. These are the values obtained by averaging the values from 12 periods of the simulation over a year (monthly regrouping times) over 1,000 iterations of the simulation and for the 5 different herds in the study. The formulated diet for 1 group had a concentration of 1.5 Mcal/kg DM. Having more groups divides the cows into more homogeneous NE_L concentration groups and hence higher and lower concentrations of NE_L in the diet. A similar pattern was observed in RDP and RUP percentages in the diet. The reported NE_L concentrations by McGilliard et al. (1983) using a clustering method with two groups were 1.62 (high) and 1.42 (low) Mcal/kg, which are comparable with the ones obtained here (1.59 and 1.41 Mcal/kg, respectively). The optimum allocation of NE_L concentration found in St-Pierre and Thraen (1999) was much less variable and higher than reported here or in the McGilliard et al. (1983) study. The optimum allocation of NE_L found in St-Pierre and Thraen (1999) study was 1.78 (Mcal/kg) in the 1 group case and remained above 1.7 even in the case of 3 groups. Previous studies have used CP for estimation of the required protein in the group, whereas this study used the MP requirement of the cows. Thus, there is no other study to compare the calculated RDP and RUP percentages, but based on author’s expertise the calculated values for these figures are in the expected range.

Table 6.5. Formulated diet components for different nutritional group numbers and scenarios obtained by averaging 5 herds (±SD within herds) throughout the simulation period (d=1 to d=365) and over 1,000 iterations.

Group number	Groups	NE_L (Mcal/kg DM)	RDP (% of DM)	RUP (% of DM)		
				0xSD	0.5xSD	1xSD
Grouping all lactating cows (DIM>21 and not dry)						
1 Group	G1	1.5±0.004	9.34±0.0002	5.06±0.0004	5.46±0.0004	5.85±0.0005
2 Groups	G1	1.59±0.005	9.89±0.0003	5.35±0.0004	5.63±0.0005	5.90±0.0005
	G2	1.41±0.005	8.83±0.0003	4.78±0.0005	5.01±0.0005	5.22±0.0006
3 Groups	G1	1.66±0.006	10.27±0.0003	5.42±0.0005	5.68±0.0005	5.95±0.0006
	G2	1.48±0.005	9.25±0.0003	5.15±0.0003	5.27±0.0005	5.36±0.0004
	G3	1.38±0.006	8.67±0.0003	4.67±0.0004	4.85±0.0006	5.02±0.0006
4 Groups[1]	G1	1.72	10.60	5.42	5.68	5.95
	G2	1.52	9.49	5.24	5.38	5.50
	G3	1.45	9.07	4.99	5.08	5.18
	G4	1.37	8.59	4.61	4.75	4.93
Separating first lactation cows from older lactating cows						
First lactation group[2]		1.5±0.008	9.34±0.0005	4.93±0.0007	5.24±0.0006	5.55±0.0005
1 Group	G1	1.5±0.003	9.35±0.0002	5.15±0.0003	5.57±0.0004	6.00±0.0005
2 Groups	G1	1.61±0.005	9.97±0.0002	5.46±0.0004	5.75±0.0005	6.03±0.0005
	G2	1.40±0.002	8.77±0.0002	4.85±0.0002	5.08±0.0002	5.31±0.0002
3 Groups	G1	1.67±0.006	10.33±0.0004	5.53±0.0005	5.80±0.0006	6.07±0.0006
	G2	1.48±0.003	9.24±0.0002	5.24±0.0003	5.35±0.0003	5.46±0.0004
	G3	1.37±0.004	8.60±0.0002	4.72±0.0003	4.90±0.0002	5.09±0.0002
4 Groups[1]	G1	1.72	10.6	5.54	5.81	6.08
	G2	1.52	9.49	5.28	5.46	5.60
	G3	1.44	9.03	4.95	5.13	5.28
	G4	1.35	8.55	4.62	4.78	4.98

[1] 4 groups were just experimented on the largest herd (1,460-cow herd)

[2] The average formulated diet for first lactation cows separated from older cows was similar across all the grouping numbers and herds

6.12.4. Captured Energy in Milk

Percentage of total NE_L consumed that was captured in milk is presented in Figure 6.11. This percentage did not show a consistent increase with more groups and the values were rather similar among different group numbers for all the herds. The similarity of the percentage of total NE_L intake that is captured in milk has also been shown in a field trial study by Smith et al. (1978). The results of the current study could be explained by studying the detailed charts of the NE_L concentration in the diet (Figure 6.12) and the distribution of the retained body energy in terms of BCS (Figure 6.13). Average NE_L and MP concentrations in the diet for all the cows in the 727-cow herd over all the lactations for 1,000 replications are illustrated in Figure 5. In the case of 1 group a greater proportion of the cows in the herd were underfed than with more groups and therefore the total NE_L consumption and milk yield (milk yield depended on the energy in the body as captured in BCS) for just 1 group was less than 2 and 3 groups. It should be noted that upper and lower bounds of BCS were restricted between 2.0 and 4.5. Light shade of lines in Figure 6.12 show that utilizing 2 or 3 groups increased the diet NE_L concentration in early lactation (the time that is most needed) until around 150 d postpartum. After this point 2 and 3 groups have a lower NE_L concentration in the diet than 1 group. The overall lower NE_L concentration required for late lactation cows was generally lower than the higher NE_L concentration required for early lactation cows and therefore their negative difference. Thus, the trend for percentage of total energy captured in milk was not consistent among herds. The sudden drop of NE_L concentration at 22 d postpartum (moving time of fresh groups to lactating groups) in 1 group should be noted. Cows in 1 group were then fed close to the average of the group NE_L concentration of their requirements (approximately 1.5 Mcal/kg DM), which remained almost unchanged until around 300 DIM. At this point the increasing proportion of dry cows and low producing late lactation cows reduced the average NE_L concentration. On the other hand, in the case of 2 and 3 groups, after 22 d postpartum, there was a curvilinear pattern, which is explained by the fact that cows were fed closer to their requirements (and at higher concentration than 1 group) when the energy requirements were high. After passing the critical point of early lactation, NE_L concentration dropped for 2 and 3 groups compared to 1 group. Two and 3 groups assure that late lactation cows have enough energy in the diet, but not much more than required. Overall, it is clear that 2 and 3 groups, distributes NE_L more efficiently based on DIM and productivity, which might increase overall NE_L consumption in the herd (Figure 6.12).

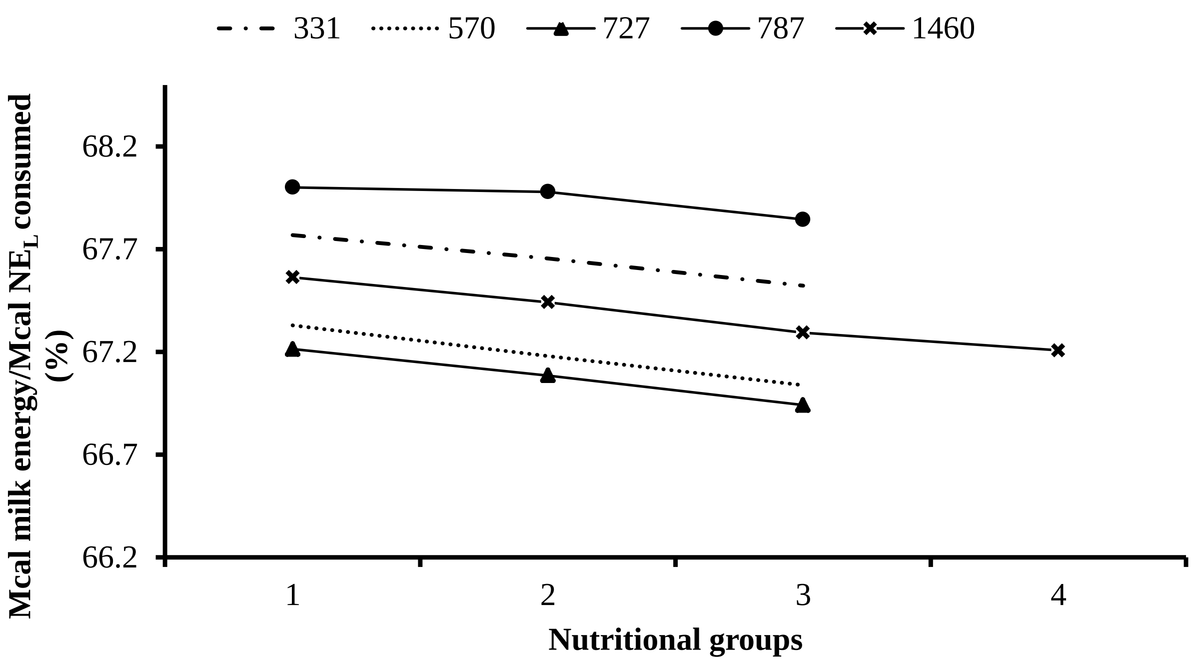

Figure 6.11. Percentage of total NE_L consumed captured in milk according to number of groups and for the 5 different herds (labeled by the number of adult cows available in the herds).

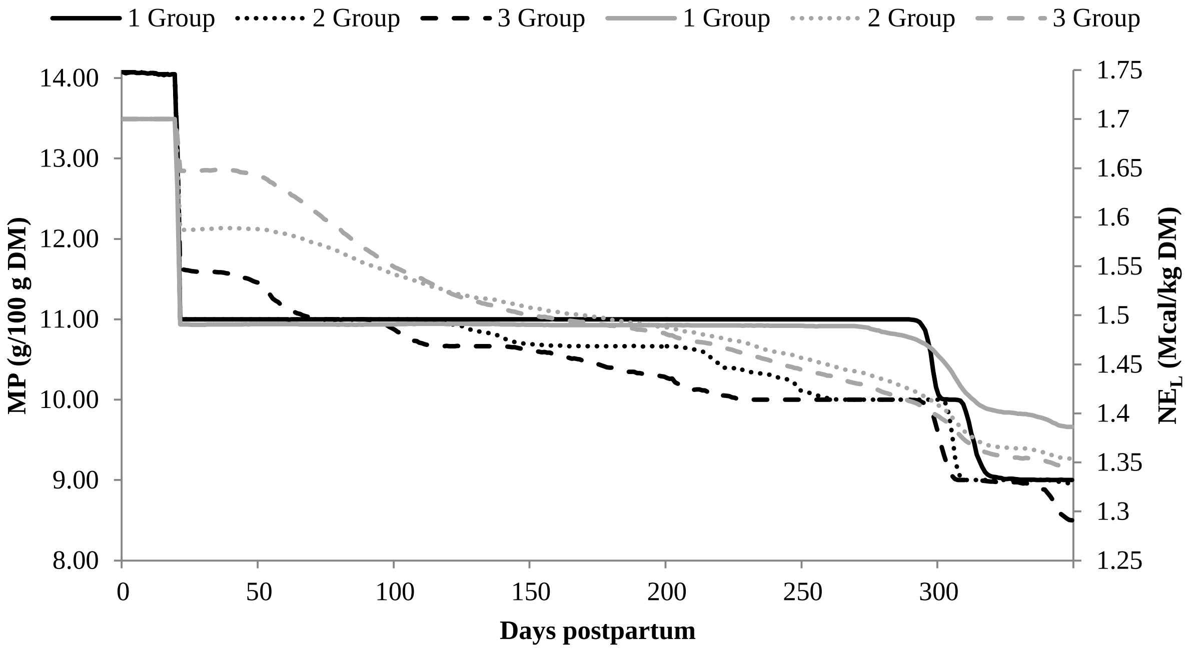

Figure 6.12. Offered diet average NE_L (light shade) and metabolizable protein (MP; dark shade) after calving for the 727-cow herd under different number of nutritional groups from 1,000 replications.

Excess energy in late lactation cows is associated with greater BCS and over conditioned cows that can have complications in the next lactation (Cameron et al., 1998). The effect of a number of nutritional groups on BW and BCS can be seen in Figure 6.13, which compares the impact of 1 and 3 nutritional groups on BW and BCS distributions of the 787-cow herd. Left panel of Figure 6.13 shows that the BW density plot of two grouping strategies (1 vs. 3 groups) does not differ considerably. They both have similar distributions. This indicates that 1 and 3 groups did not result in overall BW changes of the cows in the herds. The stable BW among different grouping numbers has also been found in field trials (Smith et al., 1978; Clark et al., 1980; Kroll et al., 1987). Right panel of Figure 6.13 illustrates the effect of nutritional grouping on the distribution of the cows' body energy content (BCS). The 1 group represented by a dark shade density plot has a different distribution than 3 groups (light shade). With 1 group the distribution is a thick-tailed, which means that the model projects many cows that are either under-conditioned (BCS = 2.0) or over-conditioned (BCS = 4.5), and it has a mode around BCS = 2.75. On the other hand, 3 groups show a rather normal distribution curve with the mode around BCS = 3.25. Similar distribution was observed in the case of 2 groups and in the other studied herds (data not shown). Having 2 or 3 groups appears to ensure that the consumed energy is better distributed promoting healthier cows.

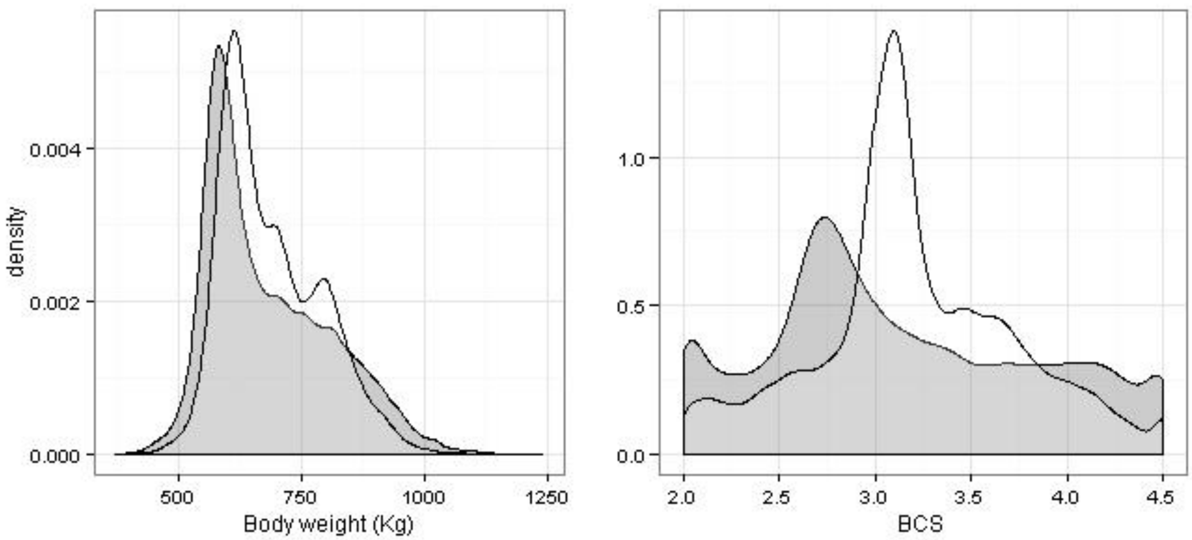

Figure 6.13. Body weight (left) and BCS (right) density plot created from aggregating all the cows from 1,000 replications from the 787-cow herd for 1 (dark shade) and 3 (light shade) nutritional groups. Total area under the curves adds to 1.

6.12.5. Captured Nitrogen in Milk

Percentage of total N consumed that was captured in milk is depicted in

Figure 6.14. It is clear that percentage of N captured in milk increases with the number of groups at different slopes. Most of the gain was captured when moving to 2 nutritional groups. To depict the source of these higher percentages of N captured in milk, dark lines in Figure 6.12 are provided. Figure 6.12 illustrates the actual MP offered in the diet (g/100 g DM) under different grouping strategies for the 727-cow herd. The overall trend is similar to the provided

NE_L in the diet, previously discussed. Until 21 d postpartum all cows were fed a high concentrate diet with 14 g/100 g DM MP regardless of the grouping strategy. After 21 d postpartum cows were moved to optional groups, which enabled the model to group them based on their requirements and number of groups under study. In the 1 group case, the MP consumption dropped to 11 g/100 g DM, stayed at the same level until about 300 d postpartum when it dropped consistently through the rest of the lactation. However, in the 2 and 3 groups the provided MP in the diet was closer to the actual requirements. Therefore with 2 or 3 groups, cows until about 100 d postpartum were fed more MP and thereafter were fed lower MP than the 1 group case. This higher N consumption in late lactation for 1 group compared to more groups is consistent with the literature (VandeHaar, 2014). Having 3 groups and formulating the diet at 1xSD above the MP average improved N efficiency by 2.7% on average. This figure in St-Pierre and Thraen (1999) was 5.8%. The reason for this discrepancy could be attributed to the differences in the modeling framework and the underlying assumptions between the 2 studies. In this study, we strived to model the whole herd throughout time, with tracking all the information regarding the groupings and changes in BW and BCS. St-Pierre and Thraen (1999) simulated the actual CP and NE_L requirements of the cows without considering the herd dynamics.

The main economic gain of having more groups could be attributed to an increased percentage of N captured in milk, which in turn decreases feed cost related to RUP. Having more groups clearly improves the percentage of N captured in milk (Figure 6.14), which, at the same time, improves environmental stewardship by decreasing the amount of N excreted (VandeHaar, 2014).

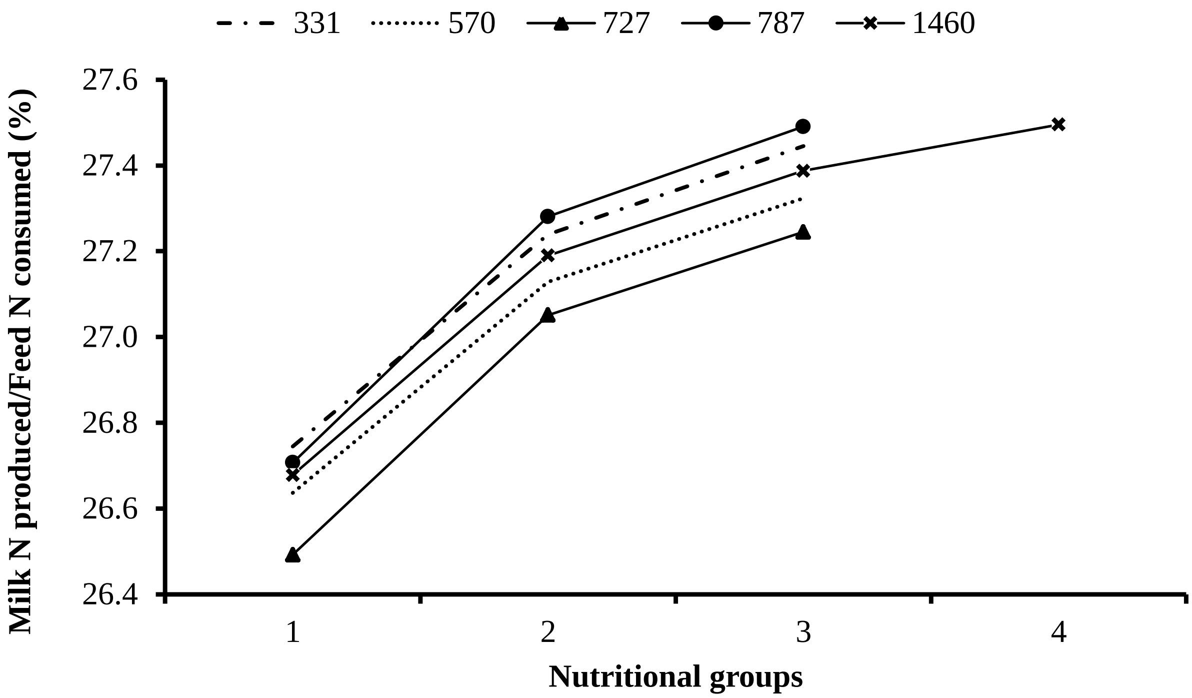

Figure 6.14. Percentage of total consumed N captured in milk according to number of groups and for 5 different herds. Label numbers represent the herd size in number of cows.

6.12.6. Scenario Analyses

Table 6.6 presents the results from scenario analyses on the input price and inclusion of milk loss and separation of the first lactating cows from older cows. The results show that even in the worst economic conditions (lowest milk price with highest nutrient costs) grouping cows had a similar average IOFC gain compared to the base scenario. Comparing the base and the best case scenarios over all the herds, the average IOFC gain ($/cow per yr) was $6 higher in 2 groups and $4 in 3 groups. Comparing the IOFC gain ($/cow per yr) of 2 and 3 groups the results showed that the relative gain was the highest in the worst case scenario ($10) and the lowest relative IOFC gain of having 3 groups instead of 2 groups was under the best case scenario($6). This emphasized the importance of grouping lactating cows in tough economic conditions, when the milk price is low compared to feed price.

Table 6.6. Economic gain in IOFC of grouping strategies under 5 studied scenarios.

Herd Size	Scenario	Difference between grouping strategies and 1 group ($/cow per yr)		
		2 Groups	3 Groups	4 Groups
	base[1]	34.6	41.8	-
331	Worst[2]	32.0	41.7	-
	Best[3]	39.3	44.2	-
	milk loss[4]	13.9	22.4	-
	1^{st} lactation[5]	31.9	38.0	-
	base[1]	32.9	39.7	-
570	Worst[2]	31.5	40.4	-
	Best[3]	36.5	41.1	-
	milk loss[4]	13.5	18.7	-
	1^{st} lactation[5]	27.2	32.7	-
	base[1]	40.6	47.9	-
727	Worst[2]	37.1	46.1	-
	Best[3]	46.6	52.5	-
	milk loss[4]	24.7	27.9	-
	1^{st} lactation[5]	32.4	37.3	-
	base[1]	48.8	58.3	-
787	Worst[2]	43.0	53.6	-
	Best[3]	58.0	66.5	-
	milk loss[4]	32.3	37.9	-
	1^{st} lactation[5]	42.0	50.1	-
	base[1]	36.4	43.5	46.86
1,460	Worst[2]	33.8	42.9	47.4
	Best[3]	41.3	46.6	48.8
	milk loss[4]	17.9	22.6	23.5
	1^{st} lactation[5]	29.7	35.7	38.5

[1] Base scenario running on the average NE_L concentration and average MP+1xSD with 10 years average annual milk price ($0.39/kg) and nutrient costs (NE_L=$0.1/Mcal, RDP=$0.18/kg, and RUP = $1.04/kg)

[2] Worst case scenario couples the lowest milk price with the highest feed price from historical 10 years annual average (Milk price=$0.29/kg, NE_L=$0.14/Mcal, RDP=$0.26/kg, and RUP=$1.52/kg)

[3] Best case scenario couples the highest milk price with the lowest feed price from historical 10 years annual average (Milk price=$0.52/kg, NE_L=$0.05/Mcal, RDP=$0.09/kg, and RUP=$0.52/kg)

[4] Adding 5 d of 1.82 kg/d milk loss for cows changing to another group under base scenario

[5] Including 1^{st} lactation cows as a separate obligatory group under base scenario. In this scenario the 1 group itself has 2 groups: 1^{st} lactating cows and $\geq 2^{nd}$ lactating cows. Thus, in addition to the number of groups for older cows one group is just for first lactation cows.

Assumed milk loss (1.82 kg/d for 5 days) due to regrouping decreased the average 5 herds IOFC of 2 groups by \$18±2 across all the herds and by \$20±1 for 3 groups when compared with the 1 group (base scenario; Table 6.6). The data showed that even under the assumption of milk loss because of regrouping there is an overall economic gain. The amount of IOFC gain (\$/cow per yr) ranged from \$14 to \$32 when comparing 1 and 2 groups and when comparing 1 with 3 groups the gain ranged from \$19 to \$38. The amount of loss depended on the number of times cows were reassigned to a different group, and it was affected by cow characteristics (i.e. milk production and DIM that determine cow requirements) and the nutrient requirement variations among the cows in the groups. The trend when having milk loss because of regrouping was consistent with the base scenario in that the largest gain was observed between 1 and 2 groups. Smith et al. (1978), in a field study, compared grouping lactating cows into 1 and 2 groups. In that study, an average of decline in milk production was found to be 2 kg/cow per day for 7 days, and this amount was also affected by the parity (less milk loss for first lactation compared to older cows). Even with this amount of milk loss, the IOFC of 2 groups was \$30/cow per year greater than 1 group, and this was a result of less concentrate fed (Smith et al., 1978). This amount of gain in IOFC is in the range of values found in this study. In another field study by Zwald and Shaver (2012) the milk loss due to change in groups was reported to be insignificant. Overall, the effects of grouping on the milk production of the cows in inconclusive (Clark et al., 1980), and based on two field studies mentioned above (Smith et al., 1978; Zwald and Shaver, 2012), it seems the assumed amount of milk loss in this study (total of 9.1 kg in 5 d) could be either underestimation or overestimation. Thus, the true amount of milk loss is unknown, and studies have shown that it could be affected by the parity (Smith et al., 1978), could vary among cows based on their DIM (Kroll et al., 1987), and possibly their characteristics. It seems to be safe to assume that not every cow might experience the same amount of loss and the duration could vary among cows based on their characteristics. However, it seems the amount of saving in the feed cost due to grouping could exceed the loss in the milk production (Smith et al., 1978; Clark et al., 1980).

Adding first lactation cows as a separate group also affected the economics of nutritional groupings and is summarized in Table 6.6. The average IOFC gain among all the herds was lower than the base scenario by \$7±2/cow per yr. This smaller gain when separating first lactation cows was mostly due to the fact that having a separate group of first lactation animals assures a closer diet tailored for those cows and older cows, similar to having a separate nutritional group. Table 6.5 summarizes the formulated diet when separating first lactation cows into their own group. Regardless of the number of groups the formulated diet of the first lactation cows was the same across different group numbers and herds. However, separating the first lactation cows into a group increased the nutrients concentration of older cows' groups' diet, thus a higher feed costs (higher RUP costs) and smaller IOFC gain in this scenario. It should be mentioned that the model did not consider the possible benefit of separating first lactating animals due to social hierarchy among the younger cows and older cows, which could results in drop in feed intake and milk production of first lactation cows (Botheras, 2007). Considering this issue could have an interaction in the reported economic gain of separately grouping first lactation cows.

6.12.7. Limitations

The model described in Chapter 6 section 1attempted to consider most of the details relevant to nutritional grouping. However, like any other model, it is an abstract version of the actual herds and therefore subject to limitations. In this study, we used NRC (2001) equations to estimate the protein and energy requirements of the cows, which is a nutrient requirement-based system (St-Pierre and Thraen, 1999). This system uses the physiological status, like milk production and pregnancy status, of the cows as inputs and produces the energy and protein requirements of the given cow (St-Pierre and Thraen, 1999). This is different from the production-response system, in which the milk production and physiological status of the cows are the output of the model driven by the amount of nutrients provided in the diet. The latter system would be preferred due to the inherent relevancy of the input and outputs in the cows. Nevertheless, in this study, we used the nutrient requirements and DMI equations from NRC (2001) to estimate the energy intake of the cows. The DMI equation adjusts DMI only based on BW, and milk energy yield, with a discount in early lactation. In this equation the diet energy density does not affect DMI, therefore elevating dietary energy density results in increased energy intake. Currently, the appropriate mathematical formulation for including dietary energy density on the DMI of the cows is not available. Therefore, here to control the energy intake of the cows the BCS of the cows were arbitrarily bounded between 2.0 and 4.5. Cows that would drop BCS below 2.0 were assumed to decrease their milk production, and the cows that would reach beyond BCS of 4.5 limited their DMI. In reality it seems there is a subtle mechanism for regulating production and DMI according to the energy balance. This behavior of the model could be improved, if the data to describe the DMI regulation based on the nutrient requirements and energy density of the diet was available.

Furthermore, the current model does not consider the effect of low (high) BCS during dry periods on the next lactation production, health issues, or reproductive performance. These effects have been reported in many studies as summarized by (Roche et al., 2009). Lower BCS is associated with less milk production and lower conception rate in the next lactation. High BCS is linked to many metabolic disorders, including fatty liver, ketosis and displaced abomasum (Cameron et al., 1998). Because of it, figures reported herein could be underestimations of the true economic and efficiency gains from nutritional grouping.

Also, the developed model regrouped all the lactating cows on a monthly basis. However, the time for regrouping itself could be triggered based on a criterion in the group to find the optimum time for regrouping. For example, Østergaard et al. (1996) used daily milk thresholds to determine the regrouping time in the simulation herds.

Additionally, the effect of separating first lactation animals in this study could also include an underestimation because separating first lactation cows from mature cows could increase the milk production of the first lactation cows (Sniffen et al., 1993) due to decreased dominance effects (adult vs. first lactation cows), which have not been included in the current study.

6.13. CONCLUSIONS

A dynamic, stochastic Monte Carlo simulation model was used to quantify the economic benefit of nutritional grouping on 5 selected dairy herds with different sizes and structures. Results showed that the majority of economic gains from nutritional grouping for lactating cows could be obtained by having 2 nutritional groups (instead of 1 group besides the fresh cow group). Adding a third group increases the gains compared to 2 groups, but on a much smaller scale, and such practice might not be justified based on the additional costs that it might incur. The IOFC gain was mainly due to higher milk production and lower RUP costs, and the gain of having more groups was even more emphasized in tough economic conditions. The effect of a possible constant milk loss when regrouping cows could have a dramatic negative impact, but even after considering such a situation, a positive economic gain was accrued. Separating first lactating cows could be considered a grouping strategy in itself.

6.14. ACKNOWLEDGMENTS

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Chapter 7

Decision Support Tools

7.1. Decision Support Systems Demonstration

7.1.1. Introduction

Decision support systems (DSS) are integrated computer programs that could be used for better-informed decision-making. The core of these systems is data from different sources, which together with analytical models and techniques create valuable information. Thus, as it was discussed in Chapter 2, DSS bridge the gap between system modeling and decision makers by providing a user-friendly environment. The main components, classifications, and development process of DSS have already been discussed in Chapter 2. This chapter provides short descriptions of different DSS tools developed for this thesis.

7.1.2. Milk Curve Fitter

Milk production is the major output of dairy herds and its prediction for economic analysis, on a cow or herd level, is critical for both researchers and dairy farmers. The milk production follows a predictable pattern that let researchers (Wood, 1967b; Ehrlich, 2011) develop different non-linear milk curve functions to predict the milk production throughout the lactation. However, there are not many options to fit herd-specific production data points to lactation curves and get these milk production functions coefficients. The introduced tool here contributes to solve this problem.

The Milk Curve Fitter is a user-friendly tool for fitting test day milk productions to user-selected milk curve function (Wood's (Wood, 1976) or MilkBot® (Ehrlich, 2011)) using the Levenberg–Marquardt algorithm. This algorithm is a numeric minimization for non-linear functions that uses an iterative approach to find the parameters that minimizes the sum of squares of the difference between the predicted parameters and the input values (finding the least square difference; (Wikipedia, 2015)). This tool is available as a stand-alone Windows based application and also as an online tool that works on any web browser. Moreover, the Milk Curve Fitter module has been integrated into the UWCURepro$ DSS, which is covered next. Following is a summarized functionality of the Milk Curve Fitter online tool, which is similar to the standalone Windows application.

The Milk Curve Fitter tool has an easy to use interface with two options for milk function curves and two options of desired unit of input for test day milk productions (Pounds or Kilograms). The model functionality has been separated into 3 separate tabs across the top of the application as follow (Figure 7.1): 1) "Curve Fitter" which can be used for uploading the test day milk production and fitting the milk curves, 2) "Daily Milk Production" which can be used for creating and exporting the predicted daily milk production using the fitted parameters, and 3) "Test Model Parameters" which can be used to visually explore the impact of changing function parameters on the predicted milk curve.

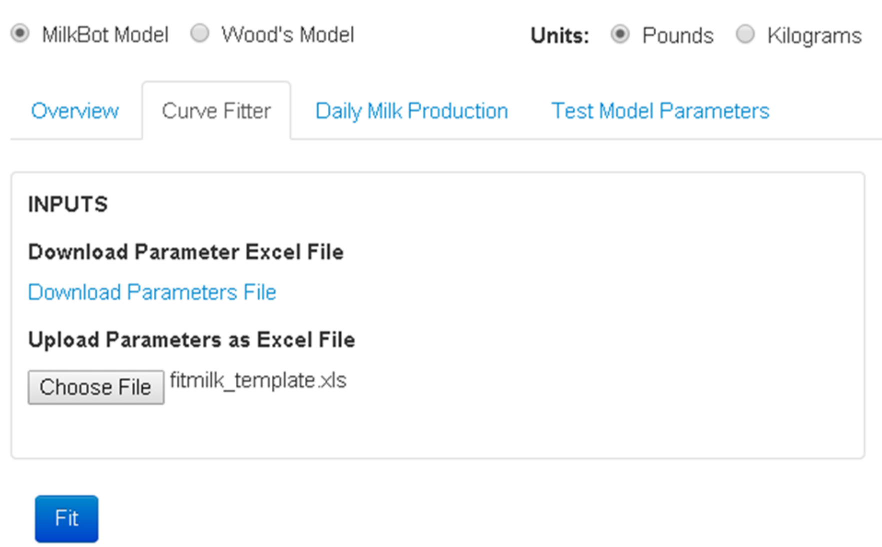
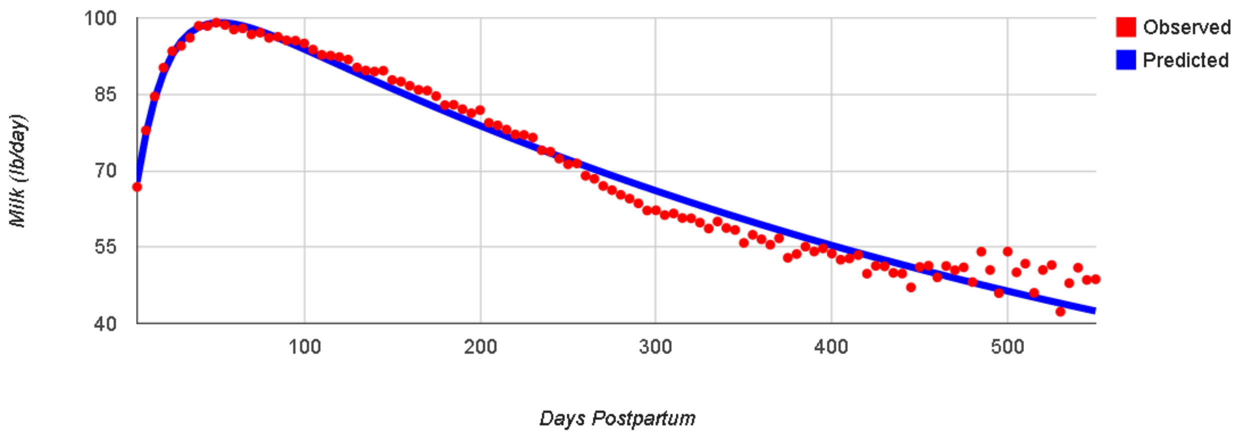


Figure 7.1. Snapshot of the online Milk Curve Fitter functionality and options separated by tabs

By running the tool (hitting Fit button), the program will perform algorithm iterations until it reaches a convergence. After the convergence, the application gives the actual milk production data points and the predicted curve using the optimized milk curve function parameters (Figure 7.2).



MilkBot® Function

$$M_{DIM} = a\left(1 - \frac{e^{\left(\frac{c-DIM}{b}\right)}}{2}\right) e^{-(d)\,(DIM)}$$

$M =$ Milk Yield
$DIM =$ Days in milk
$a =$ Scale (overall capacity to produce milk)
$b =$ Ramp (slope of milk production rising after calving)
$c =$ Offset (starting amount of milk yield)
$d =$ Decay (rate factor of decline in milk yield after peak)

OUTPUTS - Parameters Value

Parameter	Value
a	112.3355
b	18.9885
c	0.3467
d	0.0018

Figure 7.2. Snapshot of the fitted curve (blue solid line) for test day milk productions (red dots) and MilkBot® function parameters estimated by the Milk Curve Fitter.

Resulting parameters can be used to graph the lactation curve and calculate milk production for a chosen period of time (Figure 7.3). Finally, the daily milk production for a chosen period could be exported into an Excel file for further analysis.

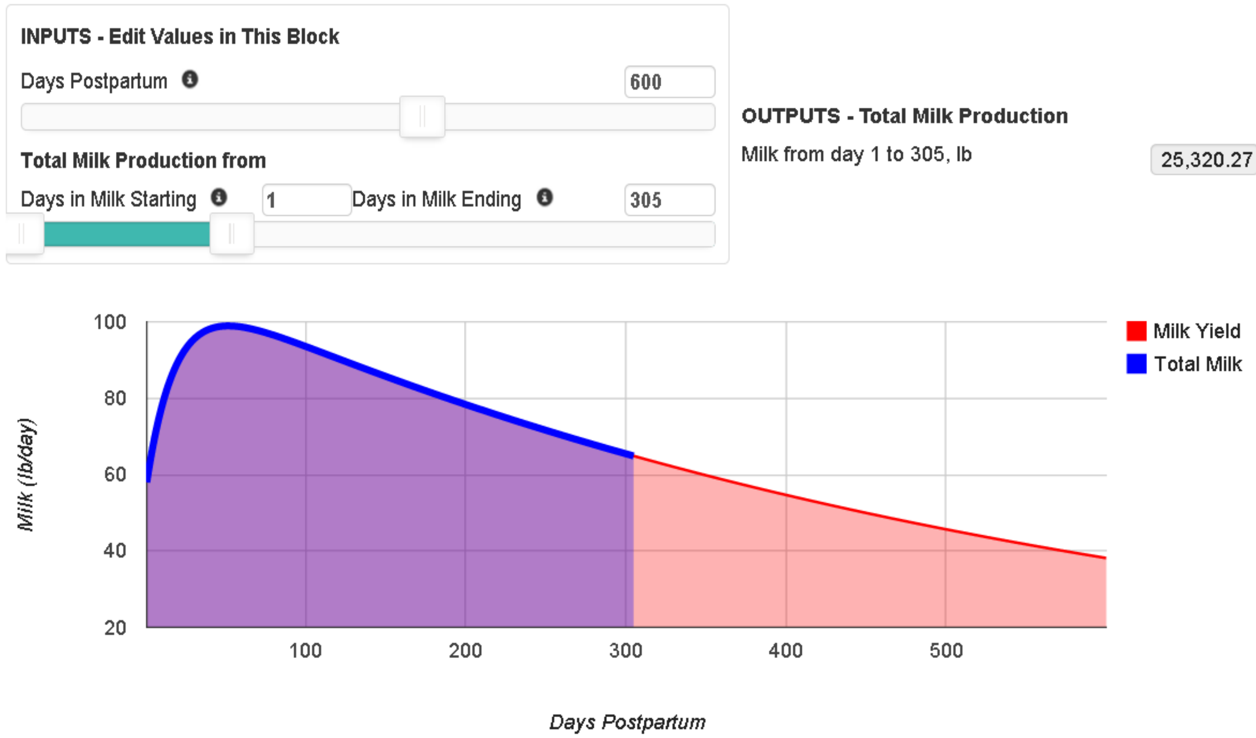


Figure 7.3. Snapshot of the "Milk Production" tab with daily milk curve fitted in Figure 7.2 until 600 days in milk. The predicted milk production between day 1 and day 305 is 25,320 lb.

Model parameters can also be manipulated in the "Test Model Parameters" tab to test their effect on the estimated curve shape and milk production. This capability is only available in the online tool.

7.1.3. Wisconsin-Cornell Dairy Repro (UWCURepro$ 1.4.0)

The great impact of reproductive performance on the dairy herd profitability is discussed in chapter 3 and 5 of this thesis. Because of its importance, numerous reproductive management strategies are constantly being developed and improved and are available for dairy farms (Giordano et al., 2012). Nonetheless, due to complex interactions and interrelations of the factors affecting reproductive performance, determining the best reproductive management under given farm's condition is challenging (Giordano et al., 2012). The developed tool in this thesis quantifies and compares the economic outcomes of common reproductive management strategies available to dairy farmers.

The daily Markov chain simulation model originally developed in Giordano et al. (2012) is used as the core for the UWCURepro$ (University of Wisconsin and Cornell University Reproductive management) DSS tool to evaluate the economic and reproductive performance of 17 common reproductive management protocols found in dairy farms in the US. Although, the daily Markov chain model (Giordano et al., 2012; summary of the model in Appendix 2) remained unchanged, the structure of the model was greatly modified to make the DSS user-friendly. This tool is a Windows based standalone application with automatic update features. The tool has been regularly updated to cover its increased demand together with the evolution of reproductive programs.

A sequential approach for acquiring data from the user is followed in this tool. All the variables in the model are divided into groups and across two tabs (Figure 7.4). The herd description tab allows the user to input parameters related to herd (e.g., herd size, rate of culling), economic (e.g., milk price, dry and lactating feed costs), and finally lactation curve data points or, alternatively, the choice from available list of pre-defined lactation curves (18,000 to 30,000 lb/cow per year). Defined lactation curves estimate daily milk production in the model. The Milk Curve Fitter defined in previous section is integrated with this tool to internally estimate the milk curve parameters of the test day milk production when entered by the user using the MilkBot® function. The next step guides the user to define reproductive management programs, which is the distinctive and unique characteristic of this tool.

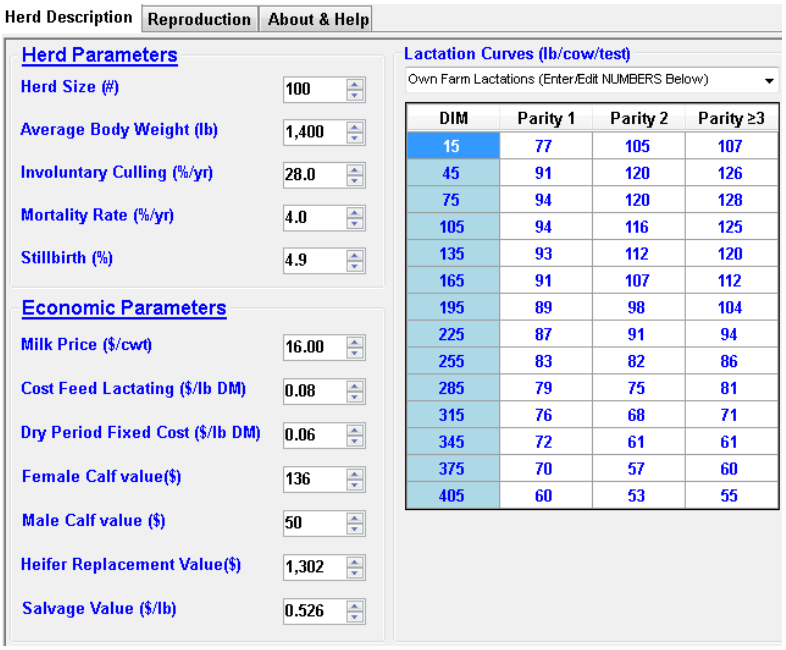


Figure 7.4. Snapshot of herd description tab from UWCURepro$ 1.4.0.

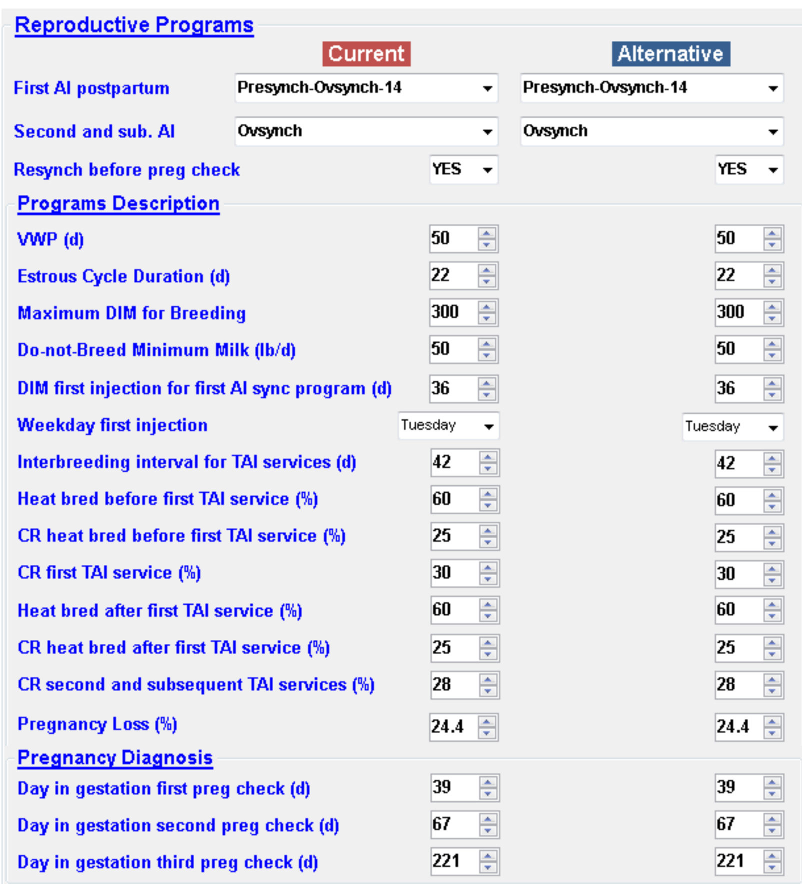


Figure 7.5. Snapshot of a section of the reproduction input tab from UWCURepro$ 1.4.0.

In the reproduction tab, every input parameter is duplicated for the current and alternative programs (Figure 7.5). This was added to this tool to give users the ability to compare different reproductive programs side by side and to explore specific reproductive alternatives that might have a better overall reproductive performance and economic return.

The user is able to run the model when all the details in the reproductive programs are provided. Due to the complexity of the model and the need to reach a steady state, the DSS takes a few minutes to complete a projection (varies depending on the number of Central Processing Units (CPU) on the computer and the defined reproductive programs), even though it uses parallel programming, which takes advantage of the power of modern multi-core processors (Ostrovsky, 2010) to minimize the total running time.

After reaching the steady-state the user can access the results tab. The results include herd economics, dynamics, and herd structure for both the current and alternative programs. For example, Figure 7.6 shows the results tab of the DSS for comparing programs using Presynch-Ovsynch 12 for the first postpartum artificial insemination (AI) service and Ovsynch for the second and subsequent AI services. The difference between these tested reproductive programs is that the alternative program uses a 100% Timed AI and the current program uses combination of estrous detection and timed AI. Panel A in this figure presents the gain/loss in the expected net revenue by changing to the alternative program. Furthermore, it shows the break-down of the economic factors involved in the overall net return (feed cost, reproductive cost, and culling cost). The DSS also reports the resulting heifer supply and demand of both reproductive programs. Panel B of Figure 7.6 presents the effect of both programs on the herd dynamics and structure.

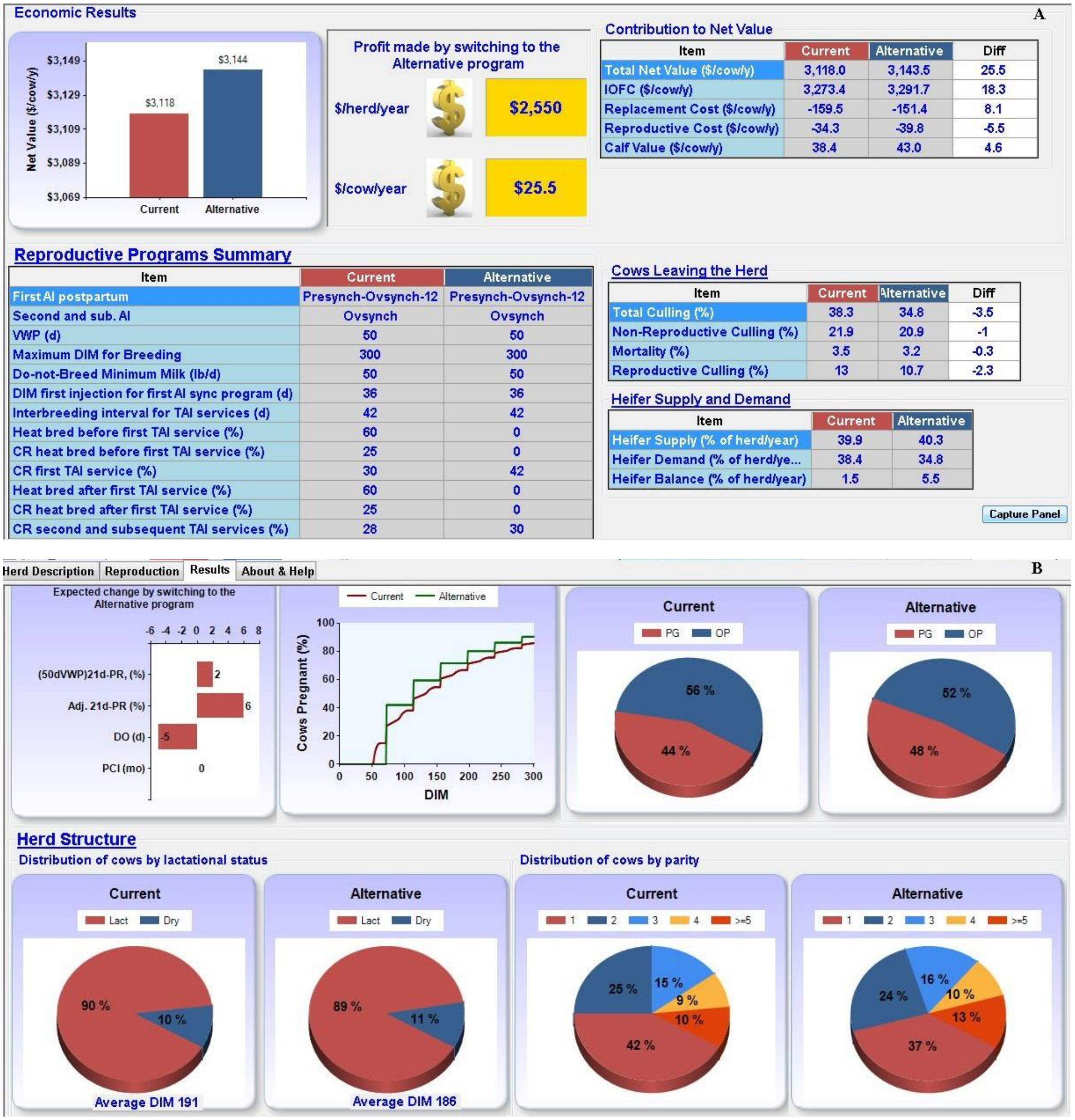


Figure 7.6. Snapshot of the results tab of UWCURepro$ 1.4.0 comparing Presynch-Ovsynch 12 for first AI and Ovsynch for second and subsequent AI services when using it combined with estrous detection (current) against using it as 100% timed AI (alternative). Panel A represents a summary of the reproductive programs used with the total expected net revenue for each program and the extra gain/loss of choosing the alternative program over the current, which in this case adds to $25.5/ cow per year. Panel B is the detail description of the herd dynamics at the steady-state, always comparing the current with an alternative.

7.1.4. Retention Pay-off Calculator

Importance of retention pay-off (RPO) in optimal culling decisions and ranking animals was discussed in chapter 3 of this thesis. It was also mentioned that dynamic programming is the state-of-the-art technique for making replacement decisions. However, dynamic programming (DP) models are computationally expensive and might not be practical for daily decision making on the farms (Shahinfar et al., 2014). The ability of machine learning algorithms for accurate predictions of non-linear and interrelated variables was used to learn the RPO values from pre-run dynamic programming model (Shahinfar et al., 2014), and was used to create the RPO DSS tool.

For this purpose, milk class (1 to 5), lactation number (1 to 9), month in milk (1 to 20), and month of pregnancy (0 to 9) can be used to describe all the cows in a DP model. The dynamic programming model (Kalantari et al., 2010; Kalantari and Cabrera, 2012) was used to run twenty-seven scenarios based on all combinations of 3 levels (base, 20% above, and 20% below) of milk production, milk price, and replacement cost. All these scenarios resulted in a dataset of 122,716 records with all the characteristics of the cows and their corresponding RPO. Then, a machine learning model tree algorithm was used to mimic the evaluated RPO from dynamic programming (Shahinfar et al., 2014). The correlation of 0.991 between the results from dynamic programming model and machine learning was obtained. This high correlation made it evident that machine learning algorithms are good candidates for predicting RPO values. Thus, an online tool was developed for calculating RPO without running the dynamic programming model (RPO calculator; Figure 7.7). This online DSS tool uses machine learning algorithms on the back-end (server side) to estimate RPO values for a given input file. The tool needs an input file from a herd with cow-level information on lactation number, days in milk, days in pregnancy, and 305 day milk mature equivalent. Given the input file, milk price, and replacement cost the tool can estimate RPO values of all the cows in the herd, instantaneously, which would not be possible by using directly the DP model.

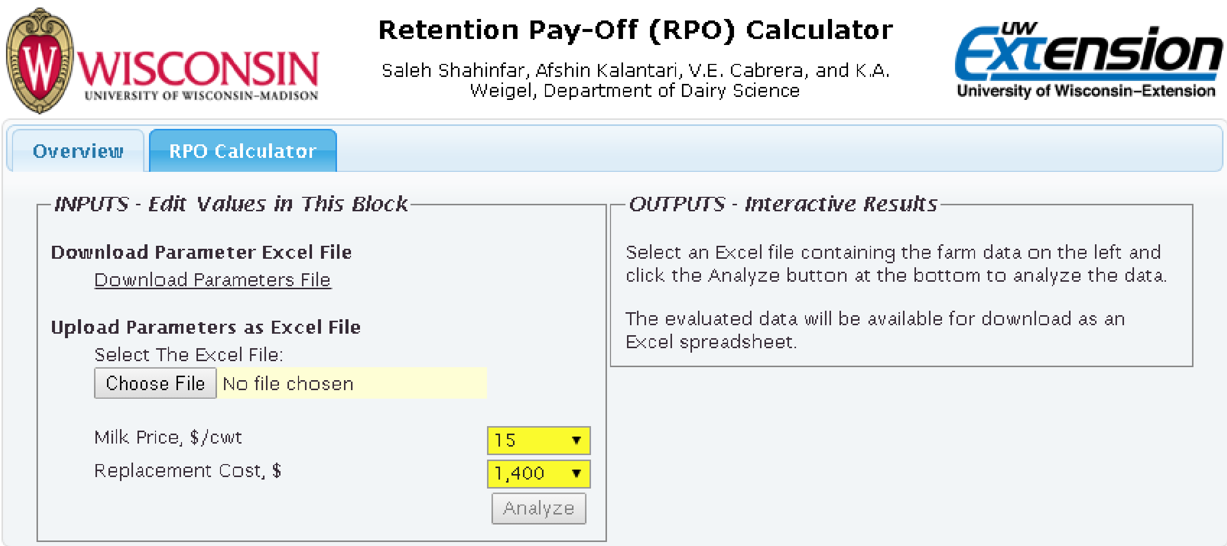


Figure 7.7. Snapshot from the on-line RPO calculator.

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Chapter 8

Conclusions

8.1. General Introduction

The overall objective of this thesis, and the included studies, was to use a subset of mathematical modeling techniques to better understand and explore the effect of changes in the input parameters of a dairy system on its output and profitability. Markov chain simulation, dynamic programming, and Monte Carlo simulation were used to: 1) evaluate the economic value of different reproductive performances and programs, 2) compare the optimal replacement decisions made by optimization and simulation techniques, 3) evaluate the economic value of nutritional grouping, and 4) develop decision support systems to assist decision makers for on-farm decisions. Here, the essence of these studies and suggestions for further improvements are summarized.

8.2. Economic Impact of Reproductive Performance

The economic impact of different reproductive performance was evaluated by two methods: 1) integrating a daily dynamic programming (DP) with a daily Markov chain simulation and 2) using robust Markov chain model to introduce the inherent uncertainty of the input parameters on the economic value generated by a given reproductive performance.

In the first study (Chapter 3) a daily DP model coupled with a Markov chain model is presented. This model was developed to evaluate the effect of different reproductive performance on the herd value. The herd value was defined as the herd's average retention payoff (RPO) weighted by the proportion of the cows at each cow state in the model. Results showed that herd values were largely influenced by reproductive performance as measured by 21-d PR. Thus, the higher the 21-d PR the higher the calculated herd value for a given reproductive program. However, an interaction was observed among the herd value, milk yield, and reproductive program. Thus, the results supported the notion of adjusting reproductive management, or specific reproductive events, according to herd milk productivity and therefore according to RPO for a better overall herd value.

The modeling framework developed in this study could be used for daily decisions of assigning cows to different reproductive management groups based on their RPO. Furthermore, there is evidence that dairy farmers could selectively breed the cows based on the probability that a given cow becomes pregnant to achieve a higher economic return (Shahinfar, 2014). These probabilities could be calculated by considering historical data on different production, reproduction, and health events from the cows (Shahinfar et al., 2015). Thus, by considering both the RPO of the cows and the probability of becoming pregnant, decision makers could potentially group the cows to either be bred or be passed over and re-evaluated at the next breeding point (Shahinfar et al., 2015). Moreover, the cows in the breeding pool can be separated based on their calculated RPO, which in turn is affected by the relative milk yield, for daily reproductive decisions such as whether to breed a heat-detected cow or not. Thus, it seems that grouping cows based on RPO and probability of becoming pregnant could be of economic importance in reproductive programs and further research in this area is warranted.

The economic impact of reproductive performance under uncertain transition probabilities, milk production and reproductive costs was evaluated using modified Markov chain model. For this purpose, a robust Markov chain model was introduced, developed, and presented in Chapter 5. The net return of a given level of 21-d PR showed a considerable variation, which was mainly due to the milk production variation among dairy farms. Due to this variability it was found that even farms with low reproductive performance could attain a good net return as long as they have a high milk production level. However, the odds of having a high net return with low 21-d

PR was low, this means that despite the milk production farmers need to strive for a better reproductive performance to ensure higher profitability. It should be noticed that, due to lack of conclusive results in the literature, the possible antagonistic (agonistic) association between milk production levels and reproductive performance were avoided (Leblanc, 2010; Bello et al., 2012). Including such possible associations could increase the model's reliability by creating estimates closer to the true economic value of improving reproductive performance. Thus, further research is needed to explore the relationship between milk production and reproduction performance.

Furthermore, introducing stochasticity into the Markov chain model produced net return distributions of the expected value for a given reproductive performance. Having distribution over model's outputs could be informative to decision makers in guiding them towards better informed decisions according to their risk preferences.

In addition, sensitivity analyses on milk price, replacement cost, and milk threshold cut-off amount indicated that the economic return associated with reproductive performance is greatly affected by the input parameters and therefore herd-specific evaluations are critical.

It should be noticed that the models presented in chapters 3 and 5 did not include health problems, such as mastitis and lameness, or seasonality, which have been shown to have considerable effect on optimal decisions, herd structure, and herd net return (Houben et al., 1994; De Vries, 2004; Cha et al., 2010). Inclusion of these important variables could dramatically improve the agreement of the model's output with the reality, thus the models could be used to study the effect of all those parameters on the final decision made on-farms. However, including these new states makes models exponentially bigger and therefore more difficult to solve.

8.3. Replacement Decisions Using: Optimization vs. Simulation

Chapter 4 of this thesis compared the recommended replacement decisions by a dynamic programming model and a Markov chain model. To make the fairest comparison, both models were fed with the same input parameters. The models were compared based on the calculated values and ranking of the cows in each model: RPO in dynamic programing model vs. cow value in Markov chain model. The observed high rank correlation of 95% for all the cow states, suggested that both models could be equally used for making replacement decisions. However, considering the model complexity, user-friendliness, and computational demands, the simulation method would be a better, more practical choice for on-farm decision-making. Furthermore, a post-optimality analysis showed a comparable herd structure by both models, and using either of the models for making culling decisions resulted in the similar net return improvement ($6/cow per year). These results strongly support that the newly developed Markov chain is a good alternative for practical dairy decision-making and for the development of decision support systems.

8.4. Economic Value of Nutritional Grouping

In Chapter 6 a dynamic, finite, stochastic Monte Carlo simulation model was developed, validated, and used to quantify the economic impact of nutritional grouping in dairy herds. For this purpose, the model was initialized by separate datasets obtained from 5 Wisconsin commercial dairy herds. The cows in the herds went through daily stochastic events and their statuses were updated over a year of running time. The effect of monthly grouping lactating cows was studied throughout a year. Cow status affected their nutrient requirements of net energy for lactation and metabolizable protein. The unique feature of the model was daily accounting of energy balance of the cows. Thus, the amount of energy consumed directly affected the BW and

BCS changes throughout lactation. Each month, lactating cows were regrouped to more homogeneous groups according to both their protein and energy concentration requirements (clustering method). Their diets were formulated based on the average and standard deviation of their nutrient concentration requirements. Results showed that the majority of income over feed costs (IOFC) gains from nutritional grouping for lactating cows could be obtained by having 2 nutritional groups (instead of 1 group besides the fresh cow group). Adding a third group increased the IOFC gains compared to 2 groups, but on a much smaller scale. The IOFC economic gains were mainly due to higher milk production and lower RUP costs. Even after considering milk loss due to regrouping the cows, a positive economic gain was obtained.

In this study, nutrient requirement-based system (NRC, 2001) was used to estimate the protein and energy requirements of the cows. This system uses the physiological status, like milk production and pregnancy status, of the cows as inputs and produces the energy and protein requirements of the given cow (St-Pierre and Thraen, 1999). This is different from the production-response system, in which the milk production and physiological status of the cows are the output of the model driven by the amount of nutrients provided in the diet. The latter system would be preferred due to the inherent relevancy of the input and outputs in the cows. Future research in nutritional grouping using production-response system could give extra insight in this multi-dimension problem.

Moreover, besides the nutrients requirements of the cows DMI equation (NRC, 2001) was used to estimate the energy intake of the cows. The DMI equation adjusts DMI only based on BW, and milk energy yield, with a discount in early lactation, diet energy density does not affect DMI, therefore elevating dietary energy density results in increased energy intake. Currently, the appropriate mathematical formulation for including dietary energy density on the DMI of the cows is not available. Therefore, here to control the energy intake of the cows the BCS of the cows were arbitrarily bounded between 2.0 and 4.5. Cows were assumed to decrease milk production at BCS of 2.0 and limit their DMI at BCS of 4.5. In reality it seems there is a subtle mechanism for regulating production and DMI according to the energy balance. This behavior of the model could be improved, if the data to describe the DMI regulation based on the nutrient requirements and energy density of the diet was available.

Furthermore, the model did not consider the effect of low (high) BCS during dry periods on the next lactation production, health issues, or reproductive performance. These effects have been reported in many studies as summarized by Roche et al. (2009). Addition of these factors would make the model more realistic and likely to increase the overall economic impact of grouping.

In this model the cows were grouped on a monthly basis, however further research is needed to find out the optimal time points at which cows could be better regrouped. Modeling the possible benefits of grouping first lactating cows separate from older cows (Sniffen et al., 1993) was ignored in this study. Addition of such details in the model could potentially add to the value generated by grouping the cows on dairy herds.

8.5. Data Challenges

Modeling dairy herds is a data intensive task and a large amount of data on different aspects of the system is often needed to model a herd. Regardless of the managerial decisions to be modeled, most of the simulation studies require data on involuntary culling, mortality, milk production, probability of pregnancy and pregnancy loss, and market prices. Availability of these data for dairy herds differs considerably and is subject to great amount of uncertainties. In the studies presented in this thesis, the national non-reproductive culling averages and reasons found in the literature (De Vries et al., 2010a; Pinedo et al., 2010) were used. The accuracy of culling data is farm dependent and varies greatly. This is due to the fact that farmers use different criteria and codes to record their health events and culling decisions. The same issue occurs for non-reproductive culling decisions of pregnant cows and the mortality rate. Regarding milk production and productivity, there are high quality data that could be used to generate lactation curves and evaluate the milk production. Also, reproductive performance measured as 21-d pregnancy rate can be easily found, which is important in modeling reproductive cycles in dairy cattle. Market prices must be the most variable input parameters, which are also affected by the region and the size of dairy herds. Among all prices, milk price and feed commodities are the most accessible. Other prices such as veterinary costs and costs of reproductive protocols are difficult to find and are highly variable. To circumvent these challenges in this thesis, sensitivity analysis was applied on the key input parameters to explore the impact of plausible changes. Sensitivity analysis and scenario building are important aspects of developing models that give an extra insight into the behavior of the underlying dairy herd system under different conditions. However, it should be noted that the results of every modeling study are highly dependent on the input data used. Thus, data should be the most accurately possible to ensure the mathematical models and decision support systems would produce reliable results for decision making.

8.6. Decision Support Systems

Decision support systems are proven to be critical in assisting decision makers. Some of the mathematical models developed for this thesis have been converted to user-friendly decision support systems for the benefit of decision makers on dairy farms. The developed tools in this thesis (Chapter 7) include: 1) “Milk Curve Fitter” for fitting lactation curves on milk test day input data, 2) "UWCURepro$" for evaluating economic gains/losses of some common reproductive management strategies in dairy farms, and 3) "RPO Calculator" for estimating RPO values and ranking cows based on their RPO.

Such research based decision support systems could be integrated into, current or future, commercial database management systems to be used for dynamic and on-farm decision-making.

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Appendices

Appendix 1

Python 2.7 is used for scripting simple examples below. The easiest method to run the scripts is to download and install Enthought Canopy (https://www.enthought.com/products/canopy/). This Integrated Development Environment (IDE) includes all the required libraries and is the easiest way to run python scripts on Windows environment.

Script 1. Implementation of the simple dairy cow replacement problem using value iteration algorithm in Python

```
import numpy as np
import matplotlib.pyplot as plt

#setting up the transition matrices and reward vectors
#5*5Transition matrix for keeping
Keep = np.array(
[[0.302,0.449,0.219,0.029,.001],
[0.117,0.386,0.383,0.106,0.008],
[0.036,0.238,0.452,0.238,0.036],
[0.008,0.106,0.383,0.386,0.117],
[0.001,0.029,0.219,0.449,0.302]],
np.float)

#5*5Transition matrix for replacing
Replace = np.array(
[[0.07,0.24,0.38,0.24,0.07],
[0.07,0.24,0.38,0.24,0.07],
[0.07,0.24,0.38,0.24,0.07],
[0.07,0.24,0.38,0.24,0.07],
[0.07,0.24,0.38,0.24,0.07]],
np.float)

#Keep Reward vector
#Rk = np.array([1725,2050,2350,2620,2977])
Rk = np.array([1700,2200,2700,3200,3700])#linear 500 Inc

transactionCost =700 #replacementcost - Calf Value - Carcass Value
#Replace reward vector
Rr = Rk - transactionCost

#Possible actions to choose from
actions = ["Keep","Replace"]

#creating dictionary from the actions and transitions
transDict = {}
transDict["Keep"] = Keep
transDict["Replace"] = Replace

#Creating dictionary from Rewards
rewardsDict = {}
```

```
   rewardsDict["Keep"] = Rk
   rewardsDict["Replace"]= Rr

   #Discount factor
   beta = 0.85

   #Number of iteration in planning horizeon
   T = 50

   def valueIteration(itr):
       # (Step 1)initializing
       Vt = np.zeros([itr,len(Keep)])
       RPO = np.zeros([itr,len(Keep)])

       #array to keep the calculated values in each iteration
       #the fst dimension for actions (keep and replace) and second for all
states
       Qsa = np.zeros([len(actions),len(Keep)],np.float)

       #an array of optimum policy at each iteration
       optPolicy = np.zeros([itr,len(Keep)],np.str)

       # (Step 2) Value iteration step
       for i in range(1,itr,1): # iteration over the planning horizen
           #for each state in the state space
           for st in range(len(Keep)):
               # for each actions
               for a in actions: #[Keep,Replace]

                   Qsa[actions.index(a),st] = rewardsDict[a][st] + beta *
sum(transDict[a][st]*Vt[i-1])

           #Choosing the maximum value from the selected actions
           optInx=np.argmax(Qsa,axis=0)#np.unravel_index(Qsa.argmax(),
Qsa.shape)#Qsa.argmax() # choosing the maximizing policy
           RPO[i] = Qsa[0]-Qsa[1]
           #iterating over the calculated values and set the optimum policy
           s=0
           for p in optInx:
               optPolicy[i,s] = actions[p]
               Vt[i,s] = Qsa[p,s]
               s+=1

       return (Vt,optPolicy,RPO)


   (Vts,opts,RPO)=valueIteration(T)

   #printing final values
   print Vts[T-1]
   print opts[T-1]
   print RPO[T-1]
```

Script 2. Implementation of simple dairy cow example of Monte Carlo simulation in Python

```
import scipy.stats as sci
import matplotlib.pyplot as plt
import numpy as np

#Simulation variables
herd = 1000 # number of herds
repl = 1000 # number of cows or replications
milkClass = 6 # 5milk classes from 1
mclass = range(1,milkClass) # Vector of all the possible milk classes
milkProbs = [0.036,0.238,0.452,0.238,0.036] #probability of being at each
milk class
averageMilk = 10000 # average milk production of the herd kg/yr
'''
Simulating milk production and feed costs of every cow in the given herd
with milkPrice and type of the milk distribution
'''
def cowSim(milkPrice,normMilk):

    #decision on either use the continuos normal distribution
    #or the 5 discrete milk classes
    if normMilk:
        milkRev = milkPrice * (sci.norm.rvs(averageMilk,1300,size=
repl)/100)
    else:
        #creating discrete distribution
        loaded = sci.rv_discrete(values=(mclass,milkProbs))
        mclassRep=loaded.rvs(size=repl)
        relativeMilkY = 0.64 +0.12 * mclassRep #Converting the milk class #
to relative milk Yield
        #Annual milk Rev
        milkRev=milkPrice * relativeMilkY*(averageMilk/100)

    feedCst = sci.norm.rvs(850,30,size=repl)
    IOFC = milkRev-feedCst

    return (IOFC)
'''
Herd simulator function
milkPriceRep = vector of milk prices
normMilk = boolean variable for deciding if use the normal milk
distribution
or discrete values of 5 milk calsses
'''
def HerdSim(milkPriceRep,normMilk):
    herdsAvg = np.empty([herd])
    oneHerd = np.empty([repl])
    #looping over number of herds and apply the simulation
    for i in range(herd):
        np.random.seed(seed=233423 + i)
        #Creating different plots for one run and multiple runs
        if i == 0:
            oneHerd = cowSim(milkPriceRep[i],normMilk)
            herdsAvg[i] = np.average(oneHerd)
        else:
            herdsAvg[i] = np.average(cowSim(milkPriceRep[i],normMilk))
```

```
        return (oneHerd,herdsAvg)

    # Milk price scenarios
    milkPriceRep = 19 + (7 * (sci.beta.rvs(0.589,0.825,size=herd)))
    milkPriceRep2 = sci.uniform.rvs(19,7,size=herd)

    herdSum = np.empty([herd,3])
    oneHerd = np.empty([repl,3])
    #Running the model for three different scenarios
    (oneHerd[:,0],herdSum[:,0]) = HerdSim(milkPriceRep,False)
    (oneHerd[:,1],herdSum[:,1]) = HerdSim(milkPriceRep2,False)
    (oneHerd[:,2],herdSum[:,2]) = HerdSim(milkPriceRep,True)

    #Creating cumulative distribution function of the IOFC
    #Plotting  all the cows of just one herd with 5 discrete milk classes
    plt.subplot(2, 1, 1)
    sortedMc = np.sort(oneHerd[:,0])
    yvals = np.arange(len(sortedMc))/float(len(sortedMc))
    plt.plot(sortedMc, yvals,marker='^',markersize=5,markevery=100)

    #Plotting all the cows of one herd with normal milk production
    sortedMn = np.sort(oneHerd[:,2])
    yvals = np.arange(len(sortedMn))/float(len(sortedMn))
    plt.plot(sortedMn, yvals,marker='o',markersize=5,markevery=100)

    #setting the legend
    plt.ylabel("Probability")
    plt.legend(('5 discrete milk ','Normal milk prod.'),loc=4, fancybox=True,
framealpha=0.5)

    #plotting the average IOFC for all the cows in a herd
    plt.subplot(2, 1, 2)
    sorted0 = np.sort(herdSum[:,0])
    yvals = np.arange(len(sorted0))/float(len(sorted0))
    plt.plot(sorted0,yvals, marker='*',markersize=3,markevery=120)

    sorted1 = np.sort(herdSum[:,1])
    yvals = np.arange(len(sorted1))/float(len(sorted1))
    plt.plot(sorted1,yvals, marker='D',markersize=5,markevery=100)

    sorted2 = np.sort(herdSum[:,2])
    yvals = np.arange(len(sorted2))/float(len(sorted2))
    plt.plot(sorted2,yvals, marker='o',markersize=5,markevery=100)

    plt.ylabel("Probability")
    plt.legend(('MP beta dist.','MP unifrom dist.', 'Normal milk prod.'),loc=4,
fancybox=True, framealpha=0.5)
    plt.xlabel("IOFC ($/cow per yr)")
    plt.show()

    #printing summary statistics
    print str(np.average(herdSum[:,0]) ) + ',' + str(np.std(herdSum[:,0]))
    print str(np.average(herdSum[:,1]) ) + ',' + str(np.std(herdSum[:,1]))
    print str(np.average(herdSum[:,2]) ) + ',' + str(np.std(herdSum[:,2]))
```

Appendix 2

Daily Markov chain model

A brief description of daily Markov chain simulation model following the notation used in Giordano et al. (2012) is presented below. A daily Markov chain model was developed to simulate a dairy herd structure and dynamics at steady-state. In this Markov chain model, cows were described using 3 state variables: parity (l=1 to 9), days in milk (**DIM**=1 to 750), and reproductive status (p=0 to 282, where 0 means non-pregnant and p=1 to 282 for days in pregnancy). The model used transition probabilities for involuntary culling, dying, pregnancy, and abortion.

Using daily steps and transition probabilities cows were distributed in the state space of approximately 600,000 possible cow states. Proportion of cows in each state depended on the transition probabilities, managerial constraints (e.g., cut-off DIM for breeding purposes, voluntary waiting period (**VWP**)), and status of the cow. Equations to find the proportion of the cows at each state at the next day regarding to the current state of the cow ($Ps_{l,DIM+1,p}$) follow:

For non-pregnant cows eligible for insemination (p = 0 and $VWP = 50 \leq d \leq cut-off = 300$) and becoming pregnant:

$$Ps_{l,DIM+1,1} = Ps_{l,DIM,0} \times (1 - PleaveO_{l,DIM}) \times Ppreg_{l,DIM}, \quad [1]$$

For non-pregnant cows eligible for insemination and not becoming pregnant:

$$Ps_{l,DIM+1,0} = Ps_{l,DIM,0} \times (1 - PleaveO_{l,DIM}) \times (1 - Ppreg_{l,DIM}), \quad [2]$$

For non-pregnant cows and not eligible for insemination:

$$Ps_{l,DIM+1,0} = Ps_{l,DIM,0} \times (1 - PleaveO_{l,DIM}), \quad [3]$$

For pregnant cows without an abortion:

$$Ps_{l,DIM+1,p+1} = Ps_{l,DIM,p} \times (1 - PleaveP_{l,p}) \times (1 - PAbort_p), \qquad [4]$$

For pregnant cows with an abortion:

$$Ps_{l,DIM+1,0} = Ps_{l,DIM,p} \times (1 - PleaveP_{l,p}) \times (PAbort_p), \qquad [5]$$

For cows calving and starting the next lactation:

$$Ps_{l+1,1,0} = \sum_{DIM=VWP+282}^{cut-off+282} Ps_{l+1,1,0} + \left[Ps_{l,DIM,282} \times \left(1 - PleaveP_{l,282}\right)\right], \qquad [6]$$

where l = parity number, DIM=days in milk, and p = days in pregnancy; $PleaveO_{l,DIM}$ and $PleaveP_{l,p}$ are the daily transition probabilities of involuntary culling of non-pregnant and pregnant cows, respectively; $Ppreg_{DIM}$ is the daily probability of pregnancy; $PAbort_p$ is the probability of pregnancy loss; cut-off is the cutoff DIM for insemination; and VWP is the voluntary waiting period in days. Detail description of the used transition probabilities and input parameters can be found in Giordano et al. (2012).

The simulation started by putting 100 similar cows (or any other number) at the first lactation, first DIM and non-pregnant (a recently calved heifer) and moved them through all defined states using the above equations. These equations were followed in an iterative manner until the proportion of cows at each state did not change from one stage to another, which is called equilibrium or steady state. The distribution of cows at steady state is often called herd structure and is affected by the used transition probabilities and managerial constraints in the model. The herd structure could be subsequently be used for calculating the net income of a herd under a given scenario or be integrated with an optimization technique to estimate the herd value. In this thesis, both of these cases have been applied and used for economic evaluation.